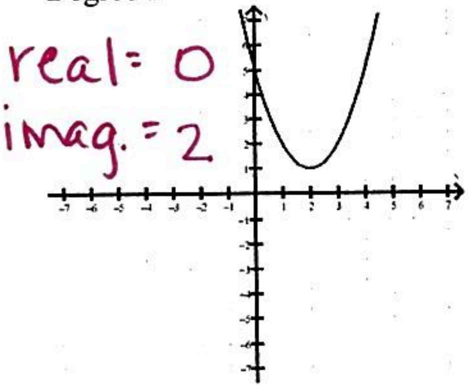
Zeros of Polynomial Functions

- Polynomials can have zeros that are real or imaginary.
- Imaginary zeros occur in PAIRS! (EX: ±2i, ±3i)
- Total # of zeros = degree of polynomial

2.5 Zeros of Polynomials

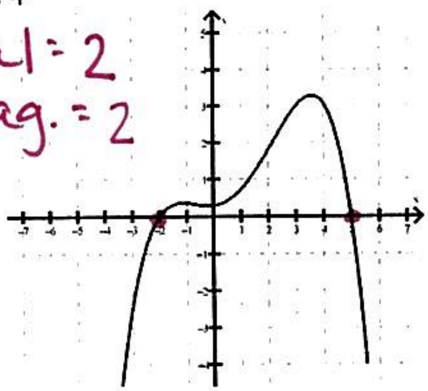
*Determine the number of real zeros and the number of imaginary zeros.

Degree 2



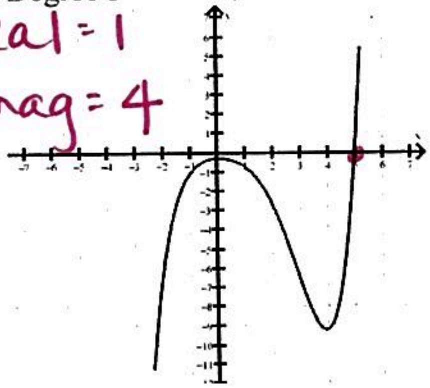
real = 0
imag. = 2

Degree 4



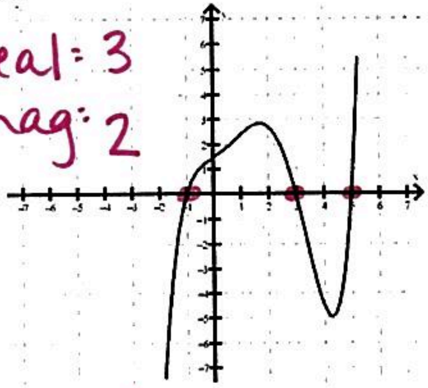
real = 2
imag. = 2

Degree 5



real = 1
imag = 4

Degree 5



real = 3
imag = 2

Can you ever have an odd number of complex roots? No!

Finding zeros:

With Calculator

- Put equation in y_1
- $y_2 = 0$
- 2nd → Trace → 5 (Intersect) → Enter 3 times

(Ex 1) $f(x) = x^4 - 3x^3 - 4x^2 + 3x + 1$

deg. 4 → 4 zeros
(+) even ↑↑

$x = -1.3194, -0.2615,$
$0.7578, 3.8231$

(Ex2) $f(x) = x^5 + 14x^4 - 10x^2 - 15$

deg. 5 → 5 zeros
(+) odd ↓↑

$x = -13.9482, -1.2429, 1.1734,$
and 2 imag.

Without Calculator

- Factor polynomial
- Use synthetic division w/ Rational Root Thm

The Rational Zeros Theorem

Let f be a polynomial function of the form $f(x) = \underset{q}{a_n} x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + \overset{p}{a_0}$ of degree $n \geq 1$ where each coefficient is an integer. If $\frac{p}{q}$ is a rational zero of f (where $\frac{p}{q}$ is written in lowest terms), then p must be a factor of the constant coefficient, a_0, and q must be a factor of the leading coefficient a_n.

CAUTION

The Rational Zeros Theorem can only provide us with a list of possible rational zeros. It **does not** guarantee that the polynomial will have a zero from the list. Simply stated, if a polynomial with integer coefficients has a rational zero, then it must be on the list created using this theorem.

EX1 $f(x) = \underset{q\ \pm 1}{1}x^3 - 5x^2 + 2x + \underset{p}{3}$ ±1, 3

Possible Rational Roots: $\frac{\pm 1, 3}{\pm 1} \rightarrow \pm 1, 3$

EX2 $f(x) = \underset{q\ \pm 1, 2}{2}x^5 - 32x^2 \underset{p}{-6}$ ±1, 2, 3, 6

Possible Rational Roots: $\frac{\pm 1, 2, 3, 6}{\pm 1, 2} \rightarrow \pm 1, \frac{1}{2}, 2, 3$
$\frac{3}{2}, 6$

EX3 Find all zeros of $f(x) = \underset{q\ \pm 1}{1}x^3 - 3x^2 + 4x \underset{p}{-12}$ ±1, 2,
4, 6

Possible Rational Roots: $\frac{\pm 1, 2, 3, 4, 6, 12}{1}$

±1, 2, 3, 4, 6, 12

Factor $x^3 - 3x^2 + 4x - 12$

	x	-3
x^2	x^3	$-3x^2$
4	$4x$	-12

$(x^2+4)(x-3) \rightarrow x = 3$

$x^2 + 4 = 0$

$\sqrt{x^2} = \pm\sqrt{-4}$ $\langle$ $\cancel{\sqrt{-1}}\ i$, $\cancel{\sqrt{4}}\ 2$

$x = \pm 2i$

Ex 4 Find all zeros of $f(x) = \underset{q}{\textcircled{1}}\underset{\pm 1}{x^3} + 3x^2 - 2x\underset{p}{\textcircled{-4}}$ ±1,2

Possible Rational Roots: $\dfrac{\pm 1,2,4}{\pm 1} = \pm 1,2,4$

* If f(x) is not factorable, you must guess & check your roots using synthetic division.

Test x=1

$$\xcancel{\begin{array}{r|rrrr} 1 & 1 & 3 & -2 & -4 \\ \oplus & \downarrow & 1 & 4 & 2 \\ \hline & 1 & 4 & 2 & \boxed{-2} \end{array}}$$

Test x=-1

$$\begin{array}{r|rrrrl} -1 & 1 & 3 & -2 & -4 & \\ \oplus & \downarrow & -1 & -2 & 4 & \\ \hline & 1 & 2 & -4 & \boxed{0} & \checkmark \\ & x^2 & x & & R & \end{array}$$

$\boxed{x=-1}$ is a root!

solve result for x $\rightarrow \textcircled{1}x^2 \textcircled{+2}x\textcircled{-4}$ Not factorable!

Use Quad. Formula ☺

$$x = \frac{-2 \pm \sqrt{\overset{4+(+16)}{2^2 - 4(1)(-4)}}}{2(1)}$$

$$\frac{-2 \pm \sqrt{20}}{2} \; \textless \; \begin{matrix}\sqrt{\cancel{4}}\ 2 \\ \sqrt{5}\end{matrix} \rightarrow \frac{-2 \pm 2\sqrt{5}}{2}$$

$\boxed{-1 \pm \sqrt{5}}$

$\boxed{\text{Zeros: } -1 \pm \sqrt{5},\ -1}$

Ex5 Find all zeros and graph

$$f(x) = \boxed{1}x^{\overset{\pm 1}{4}} - 4x^3 + 3x^2 + 4x \boxed{-4} \quad \pm 1, 2, 4$$

Possible Rational Roots : $\frac{\pm 1, 2, 4}{\pm 1} \rightarrow \pm 1, 2, 4$

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 3 & 4 & -4 \\ + & \downarrow & 1 & -3 & 0 & 4 \\ \hline & 1x^3 & -3x^2 & 0x & 4 & \lfloor 0 \; R \end{array} \checkmark$$

$\boxed{x = 1}$ is root!

* Guess + check until you get a quadratic.

$$\begin{array}{r|rrrr} -1 & \downarrow & -1 & 4 & -4 \\ + & & & & \\ \hline & 1x^2 & -4x & 4 & \lfloor 0 \; R \end{array} \checkmark$$

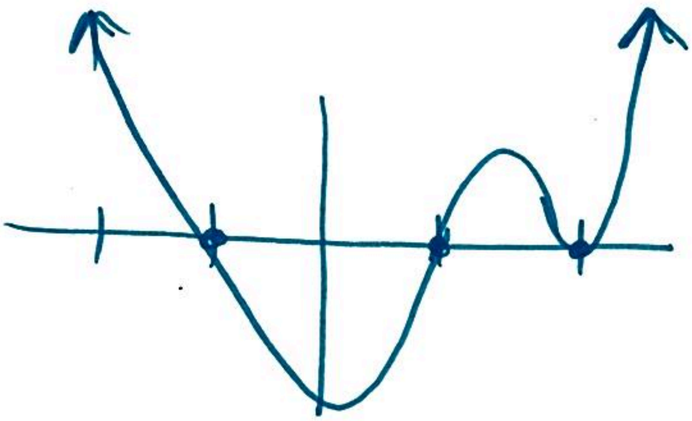
$\boxed{x = -1}$ is a root!

Solve results

$$x^2 - 4x \boxed{+4} \rightarrow (x-2)(x-2)$$

or $(x-2)^2 \rightarrow \boxed{x = 2}$ Double Root

$$\boxed{\text{Zeros: } x = 1, -1, 2}$$



(+) even ↑↑

Factored Form

$$(x-1)^1(x+1)^1(x-2)^2$$

Complex Zeros Occur in Conjugate Pairs

If a + bi where $b \neq 0$ is a zero of a function, then the conjugate a – bi is also a zero of the function.

Example 1: Find a fourth degree polynomial with real coefficients that has -1, -1, and 3i as zeros. Write in factored form and in ~~expanded~~ form.

standard

double root

and $-3i$

Factors: $(x+1)^2(x-3i)(x+3i)$

$x^2 - 9\cancel{i^2}(-1)$

Factored Form: $\boxed{(x+1)^2(x^2+9)}$

$\underbrace{(x+1)(x+1)}$

$(x^2+2x+1)(x^2+9)$

	x^2	$2x$	1
x^2	x^4	$2x^3$	x^2
9	$9x^2$	$18x$	9

Standard Form:

$\rightarrow \boxed{x^4+2x^3+10x^2+18x+9}$