

# Verifying Trigonometric Equations HW

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1)  $\sec^4 x - \tan^4 x = 2\tan^2 x + 1$

$$(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$$

$$\tan^2 x + 1 + \tan^2 x$$

$$2\tan^2 x + 1 = 2\tan^2 x + 1 \quad \checkmark$$

2)  $\tan x \sin x = \sec x - \cos x$

$$= \frac{1}{\cos x} + \frac{\cos x}{1} (\cos x)$$

$$= \frac{1 + \cos^2 x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \frac{\sin x}{\cos x} \cdot \sin x$$

$$\tan x \sin x = \tan x \sin x \quad \checkmark$$

3)  $(1 - \cos^2 a)(1 + \cos^2 a) = 2\sin^2 a - \sin^4 a$

$$\sin^2 x (1 + \cos^2 x)$$

$$\sin^2 x (1 + 1 - \sin^2 x)$$

$$\sin^2 x (2 - \sin^2 x)$$

$$2\sin^2 x - \sin^4 x = 2\sin^2 x - \sin^4 x \quad \checkmark$$

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4)  $\frac{1 - \cos x}{1 + \cos x} = (\cot x - \csc x)^2$

$$= \cot^2 x - 2\cot x \csc x + \csc^2 x$$

$$= \frac{\cos^2 x}{\sin^2 x} - 2 \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} + \frac{1}{\sin^2 x}$$

$$= \frac{\cos^2 x - 2\cos x + 1}{\sin^2 x}$$

$$= \frac{(\cos x - 1)(\cos x - 1)}{1 - \cos^2 x}$$

$$\frac{(-1)(1 - \cos x)(-1)(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}$$

$$\frac{1 - \cos x}{1 + \cos x} = \frac{1 - \cos x}{1 + \cos x} \quad \checkmark$$

5)  $\frac{1}{\sin \theta \cos \theta} - \frac{\cos \theta}{\sin \theta} = \frac{\sin \theta \cos \theta}{1 - \sin^2 \theta}$

$$= \frac{1 - \cos^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{\sin^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{\sin \theta \cos \theta}{\sin \theta \cos \theta}$$

$$= \frac{\sin \theta \cos \theta}{\cos \theta \cos \theta}$$

$$= \frac{\sin \theta \cos \theta}{\cos^2 \theta}$$

$$= \frac{\sin \theta \cos \theta}{1 - \sin^2 \theta}$$

$$= \frac{\sin \theta \cos \theta}{1 - \sin^2 \theta} = \frac{\sin \theta \cos \theta}{1 - \sin^2 \theta} \quad \checkmark$$

$$6) \frac{\sin \theta \cot \theta + \cos \theta}{2 \cot \theta} = \sin \theta$$

$$= \frac{\sin \theta \cdot \frac{\cos \theta}{\sin \theta} + \cos \theta}{\frac{2 \cos \theta}{\sin \theta}}$$

$$= \frac{2 \cos \theta \cdot \sin \theta}{2 \cos \theta}$$

$$\sin \theta = \sin \theta \quad \checkmark$$

$$7) (1 + \tan \theta) \left( \frac{\sin \theta}{\sin \theta + \cos \theta} \right) = \tan \theta$$

$$\left( \frac{1 + \sin \theta}{\cos \theta} \right) \left( \frac{\sin \theta}{\sin \theta + \cos \theta} \right)$$

$$= \left( \frac{\cos \theta + \sin \theta}{\cos \theta} \right) \left( \frac{\sin \theta}{\sin \theta + \cos \theta} \right)$$

$$= \frac{\sin \theta}{\cos \theta}$$

$$= \tan \theta = \tan \theta \quad \checkmark$$

$$8) \cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x) \cos x$$

$$= \sin^2 x (1 - \sin^2 x) (\cos x)$$

$$= \sin^2 x (\cos^2 x) (\cos x)$$

$$\cos^3 x \sin^2 x = \cos^3 x \sin^2 x \quad \checkmark$$

$$9) \frac{\sin x}{1 + \cos x} + \frac{\cot x}{1(1 + \cos x)} = \csc x$$

$$= \frac{\sin x + \cot x + \cos x \cot x}{1 + \cos x}$$

$$= \frac{\sin x + \frac{\cos x}{\sin x} + \cos x \cdot \frac{\cos x}{\sin x}}{1 + \cos x}$$

$$= \frac{\sin^2 x + \cos x + \cos^2 x}{\sin x (1 + \cos x)}$$

$$\frac{1 + \cos x}{\sin x} \rightarrow \frac{1 + \cos x}{\sin x} \cdot \frac{1}{1 + \cos x}$$

$$10) 2 \cos^3 x - \cos x = \frac{\cos^2 x - \sin^2 x}{\sec x}$$

$$= \frac{\cos^2 x (1 - \sin^2 x)}{\frac{1}{\cos x}}$$

$$= \cos^2 x - 1 + \cos^2 x$$

$$= \frac{(2 \cos^2 x - 1) \cdot \cos x}{1}$$

$$2 \cos^3 x - \cos x = 2 \cos^3 x - \cos x \quad \checkmark$$

$\csc x = \csc x$   
 $\frac{1}{\sin x} = \csc x$