PreAP Precalculus
Unit 4 Test Review

Name: Key

SEQUENCES AND SERIES

ARITHMETIC	**GEOMETRIC**
Explicit: $a_n = dn + a_0$ or $a_n = a_1 + d(n-1)$	**Explicit:** $a_n = a_1 \cdot r^{n-1}$ or $a_n = a_0 \cdot r^n$
Summation: $S_n = \frac{n}{2}(a_1 + a_n)$	**Finite Summation:** $S_n = \frac{a_1(1-r^n)}{1-r}$
	Infinite Summation: $S = \frac{a_1}{1-r}$

1) Write the first five terms and then write the explicit equation for the following sequences…

$-\frac{1}{2}, \frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \frac{15}{2}$ A) $a_1 = \frac{-1}{2}, d = 2$

$a_n = -\frac{1}{2} + 2(n-1)$

$\frac{2}{3}, -\frac{1}{3}, \frac{1}{6}, -\frac{1}{12}, \frac{1}{24}$ B) $a_1 = \frac{2}{3}, r = \frac{-1}{2}$

$a_n = \frac{2}{3}\left(\frac{-1}{2}\right)^{n-1}$

2) Given $a_2 = 9$ and $a_{k+1} = a_k - 3$, find a_5.

Recursive: previous − 3

12, 9, 6, 3, 0
(a_1, a_2, …, a_5)

$a_5 = 0$

#3-6, Find the explicit equation and then find the indicated term.

58 3) If the 3rd term of an arithmetic sequence is 94 and the 6th term is 85, find the 15th term.

n	a_n
1	100
2	97
3	94
6	85

3 < (3 to 6), > −9; ↓ +3

$d = \frac{-9}{3} = -3$

$a_1 = 100$

$a_n = 100 - 3(n-1)$

$a_{15} = 100 - 3(15-1)$

$= 58$

$-2/81$ 4) If the 4th term of a geometric sequence is -18 and the 7th is $\frac{2}{3}$, find a_{10}

n	a_n
1	486
2	−162
3	54
4	−18
7	2/3

3 < (4 to 7), > $* -\frac{1}{27}$; ↓ $* -3$

$\sqrt[3]{r^3} = \sqrt[3]{-\frac{1}{27}}$

$r = -\frac{1}{3}$

$a_n = 486\left(-\frac{1}{3}\right)^{n-1}$

$a_{10} = 486\left(-\frac{1}{3}\right)^{10-1}$

$= -\frac{2}{81}$

-217

5) If the 4th term of an arithmetic sequence is 27 and the 9th term is 7, find a_{65}

n	a_n
1	39
2	35
3	31
4	27
9	7

$d = \frac{-20}{5} = -4$

$a_n = 39 - 4(n-1)$

$a_{65} = 39 - 4(65-1)$

$= -217$

524,288

6) If the 3rd term of a geometric sequence is -32 and the 6th term is 2048, find the 10th term.

n	a_n
1	-2
2	8
3	-32
6	2048

$\sqrt[3]{r^3} = \sqrt[3]{-64}$

$r = -4$

$a_1 = -2$

$a_n = -2(-4)^{n-1}$

$a_{10} = -2(-4)^{10-1}$

$= 524,288$

#7-8, Find the indicated partial sum of the following sequences.

32

7) Find $\sum_{i=0}^{3}(2i+5)$

155/16

8) $\sum_{n=1}^{5} 5\left(\frac{1}{2}\right)^{n-1}$

#9-13, Find the indicated partial sum of the following sequences.

1100

9) 40, 37, 34, 31, … n = 25

a_1, $-3 \longrightarrow$

$\sum_{n=1}^{25}(40-3(n-1))$

$= 100$

12,355

10) If $a_n = 5n-1$ find S_{70}

$\sum_{n=1}^{70}(5n-1) = 12355$

1,793,613

11) If $a_n = \frac{1}{2}(-3)^n$ find S_{14}

$\sum_{n=1}^{14}\left(\frac{1}{2}(-3)^n\right)$

$= 1,793,613$

7200

12) Find S_{60} of the series 2, 6, 10, 14, …

a_1, $+4$ $+4 \longrightarrow$

$a_n = 2+4(n-1)$

$\sum_{n=1}^{60}(2+4(n-1))$

$= 7200$

305/81

13) Find S_5 for $5, \frac{-5}{3}, \frac{5}{9}, \frac{-5}{27}, \cdots$

$\times -\frac{1}{3}$ $\times -\frac{1}{3}$

$a_1 = 5$

$r = -\frac{1}{3}$

$\sum_{n=1}^{5}\left(5\left(-\frac{1}{3}\right)^{n-1}\right) = \frac{305}{81}$

14) A paper manufacturer buys a machine for $120,000. At the end of each year the depreciated value will be 70% of what it was at the beginning of the year. Find the depreciated value of the machine after 5 full years.

geometric

$r = 0.7$

$a_1 = 120{,}000$

$a_n = 120{,}000(0.7)^{n-1}$

$a_5 = 120{,}000(0.7)^{5-1} = \boxed{\$28{,}812}$

#15-18, Find the sum of the infinite geometric series.

15) $\sum_{i=1}^{\infty}\left(\frac{7}{8}\right)^{i-1}$

$0 < r < 1$

$a_1 = 1$

$r = 7/8$

$\frac{1}{1-7/8} = \boxed{8}$

16) $\sum_{i=1}^{\infty}(0.1)^{i-1}$

$a_1 = 1$

$r = 0.1$

$\frac{1}{1-0.1} = \boxed{\frac{10}{9}}$

17) $\sum_{k=1}^{\infty}4\left(\frac{2}{3}\right)^{k-1}$

$a_1 = 4$

$r = 2/3$

$\frac{4}{1-2/3} = \boxed{12}$

18) $\sum_{i=1}^{\infty}-4\left(\frac{5}{2}\right)^{n}$

$r > 1$

$\boxed{\text{DNE}}$

#19-20, Use Pascal's Triangle to expand and simplify the expression.

19) $(x+4)^4$ — use row 4

Coeff	1st	2nd	Term
1	x^4	$4^0 = 1$	x^4
4	x^3	$4^1 = 4$	$16x^3$
6	x^2	$4^2 = 16$	$96x^2$
4	$x^1 = x$	$4^3 = 64$	$256x$
1	$x^0 = 1$	$4^4 = 256$	256

$\boxed{x^4 + 16x^3 + 96x^2 + 256x + 256}$

20) $(a-3b)^5$ use row 5

Coeff	1st	2nd	Term
1	a^5	$(-3b)^0 = 1$	a^5
5	a^4	$(-3b)^1 = -3b$	$-15a^4b$
10	a^3	$(-3b)^2 = 9b^2$	$90a^3b^2$
10	a^2	$(-3b)^3 = -27b^3$	$-270a^2b^3$
5	$a^1 = a$	$(-3b)^4 = 81b^4$	$405ab^4$
1	$a^0 = 1$	$(-3b)^5 = 243b^5$	$243b^5$

$\boxed{a^5 - 15a^4b + 90a^3b^2 - 270a^2b^3 + 405ab^4 + 243b^5}$