Unit 3 Test Review Name KEY

Graph

1. $f(x)=2^{x+3}-4$

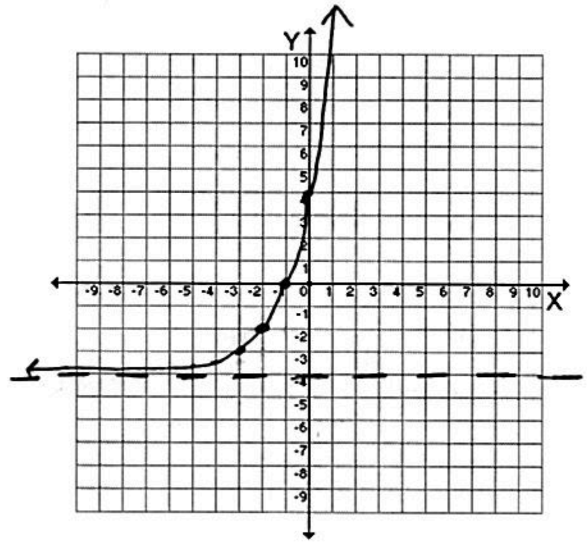
Domain: $\mathbb{R}$

Range: $y>-4$

Y-Intercept: $(0,4)$

Asymptotes: $y=-4$

x	y
-1	0
0	4
1	12



2. $f(x)=\frac{1}{2}(3)^x-3$

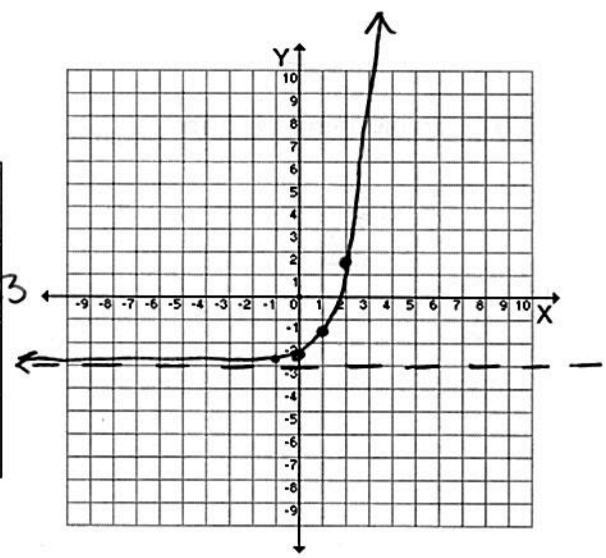
Domain: $\mathbb{R}$

Range: $y>-3$

Y-Intercept: $(0,-2.5)$

Asymptotes: $y=-3$

x	y
-1	-2.833
0	$-5/2$
1	$-3/2$



3. $f(x)=-\log_{10}(x)+2$

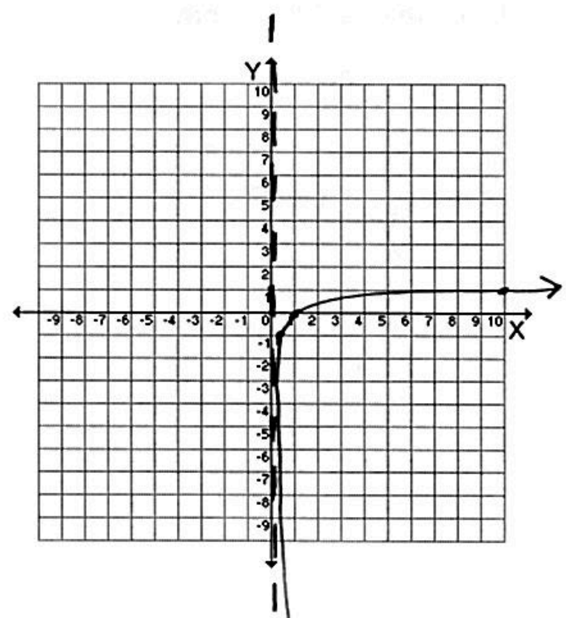
Domain: $x>0$

Range: $\mathbb{R}$

Y-Intercept: None

Asymptotes: $x=0$

x	y
0.1	-1
1	0
10	1



10^x

x	y
-1	0.1
0	1
1	10

Solve for x using same base.

4. $9^{4x} = 8$ Common Base Not Possible! $\frac{\log_9(8)}{4} = \frac{\cancel{4}x}{\cancel{4}}$ $\boxed{x = \frac{\log_9(8)}{4}}$	5. $25^{x-2} = 5^{4x+12}$ $(5^2)^{x-2} = 5^{4x+12}$ $\cancel{5}^{2x-4} = \cancel{5}^{4x+12}$ $2x-4 = 4x+12$ $-2x-12 \quad -2x-12$ $\frac{-16}{2} = \frac{\cancel{2}x}{\cancel{2}}$ $\boxed{x=-8}$	6. $27(3)^x = 243$ $\frac{\cancel{27}(3)^x}{\cancel{27}} = \frac{243}{27}$ $3^x = 9$ $\cancel{3}^x = \cancel{3}^2$ $\boxed{x=2}$
7. $3^{x+4} = \frac{1}{81}$ $3^{x+4} = 81^{-1}$ $\cancel{3}^{x+4} = (\cancel{3}^4)^{-1}$ $x+\cancel{4} = -4$ $-\cancel{4} \quad -4$ $\boxed{x=-8}$	8. $5^{r+2} \bullet 5^{2r} = 25$ add exponents! $\cancel{5}^{3r+2} = \cancel{5}^2$ $3r+2 = 2$ $-2 \quad -2$ $\frac{\cancel{3}r}{\cancel{3}} = \frac{0}{3}$ $\boxed{r=0}$	9. $16^{2-x} = \left(\frac{1}{64}\right)^{3-x}$ $16^{2-x} = (64^{-1})^{3-x}$ $(4^2)^{2-x} = (4^{-3})^{3-x}$ $\cancel{4}^{4-2x} = \cancel{4}^{-9+3x}$ $4-2x = -9+3x$ $+9+2x \quad +9+2x$ $\frac{13}{5} = \frac{\cancel{5}x}{\cancel{5}}$ $\boxed{x = \frac{13}{5}}$

Write in exponential form.

10. $\log_2\left(\frac{1}{8}\right) = -3$ $\boxed{2^{-3} = \frac{1}{8}}$	11. $\log 100 = 2$ (10) $\boxed{10^2 = 100}$	12. $\ln 20.086 = 3$ (e) $\boxed{e^3 = 20.086}$

Write in logarithmic form.

13. $10^3 = 1000$ $\log_{10}(1000) = 3$ OR $\log(1000) = 3$	14. $e^0 = 1$ $\ln_e(1) = 0$ OR $\ln(1) = 0$	15. $4^3 = 64$ $\boxed{\log_4(64) = 3}$

Use the properties of logarithms to expand the expression.

16. $\log_3 xy$

$\boxed{\log_3(x) + \log_3(y)}$

17. $\log_4 rst$

$\boxed{\log_4(r) + \log_4(s) + \log_4(t)}$

18. $\log_b \frac{\sqrt{x}}{p}$ $x^{1/2}$

$\log_b(\sqrt{x}) - \log_b(p)$

$\boxed{\frac{1}{2}\log_b(x) - \log_b(p)}$

19. $\log_3 5\sqrt[3]{a}$ $a^{1/3}$

$\log_3(5) + \log_3(\cancel{\sqrt[3]{a}})$

$\boxed{\log_3(5) + \frac{1}{3}\log_3(a)}$

Condense the expression to the logarithm of a single quantity.

20. $2\log_3 x + \frac{1}{3}\log_3 y$ $\sqrt[3]{y}$

$\log_3(x^2) + \log_3(y^{1/3})$

$\boxed{\log_3(x^2 \cdot \sqrt[3]{y})}$

21. $2\log x - 7\log y$

$\log(x^2) - \log(y^7)$

$\boxed{\log\left(\frac{x^2}{y^7}\right)}$

Evaluate. (Give simplified numerical answer).

22. $\log 1000 = x$ (10)

$10^x = 1000$

$\boxed{x = 3}$

23. $\log_9 27 = x$

$9^x = 27$

$\cancel{3}^{2x} = \cancel{3}^3$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{3}{2}$

$\boxed{x = 3/2}$

24. $\cancel{\ln e^7}$

$\boxed{7}$

25. $\log_2 \frac{1}{16} = x$

$2^x = \left(\frac{1}{16}\right)$

$\cancel{2}^x = \cancel{2}^{-4}$

$\boxed{x = -4}$

26. $e^{\ln 8 - \ln 4}$

$e^{\ln\left(\frac{8}{4}\right)}$

$\cancel{e^{\ln}}(2)$

$\boxed{2}$

27. $\log_3(\log_4 64)$

$\log_3(\log_4(4^3))$

$\log_3(3\cancel{\log_4(4)})$

$\log_3(3) = \boxed{1}$

Solve. (You may use a calculator to check your answers, but make sure that you work them by hand.)

28. $\log x + \log(x-3) = 1$ $\log(x(x-3)) = 1$ 10 $10^1 = x^2 - 3x$ $x^2 - 3x - 10 = 0$ $(x-5)(x+2) = 0$ $\boxed{x=5}$ $\cancel{x=-2}$	29. $\log_3(x-2) + \log_3 10 = \log_3(x^2+3x-10)$ $\cancel{\log_3}(10(x-2)) = \cancel{\log_3}(x^2+3x-10)$ $10x - 20 = x^2 + 3x - 10$ $-10x + 20 \mid -10x + 20$ $x^2 - 7x + 10 = 0$ $(x-2)(x-5) = 0$ $\cancel{x=2}$ $\boxed{x=5}$	30. $\log_2(4-5x) = 2$ $2^2 = 4 - 5x$ $-4 \mid -4$ $\frac{0}{-5} = \frac{-5x}{-5}$ $\boxed{x=0}$	31. $\log_5(5s) = \log_5(3-2s)$ $5s = 3 - 2s$ $+2s \mid +2s$ $\frac{7s}{7} = \frac{3}{7}$ $\boxed{s = 3/7}$
32. $2\log_2 3 - \log_2(x+1) = 3$ $\log_2\left(\frac{9}{x+1}\right) = 3$ $\frac{2^3}{1} = \frac{9}{x+1}$ $8x + 8 = 9$ $-8 \mid -8$ $8x = 1$ $\boxed{x = \frac{1}{8}}$	33. $\log_3(x^2-9) - \log_3(x+3) = 1$ $\log_3\left(\frac{(x+3)(x-3)}{x+3}\right) = 1$ $\log_3(x-3) = 1$ $3^1 = x - 3$ $+3 \mid +3$ $\boxed{6 = x}$	34. $\log_5(8-3n) = 0$ $5^0 = 8 - 3n$ $1 = 8 - 3n$ $-8 \mid -8$ $\frac{-7}{-3} = \frac{-3n}{-3}$ $\boxed{n = 7/3}$	35. $\log_6(y+4) + \log_6(3y) = 2$ $\log_6(3y(y+4)) = 2$ $6^2 = 3y^2 + 12y$ $3y^2 + 12y - 36 = 0$ $3(y^2 + 4y - 12) = 0$ $3(y+6)(y-2) = 0$ $\cancel{y=-6}$ $\boxed{y=2}$
36. $5^n = 75$ $\log_5(75) = n$ $\boxed{n = 2.682}$	37. $3^{2x+1} = 15$ $\log_3(15) = 2x + 1$ $-1 \mid -1$ $\frac{\log_3(15) - 1}{2} = \frac{2x}{2}$ $\boxed{x = 0.732}$	38. $2^x = 81$ $\log_2(81) = x$ $\boxed{x = 6.339}$	39. $e^{2x} = 5$ $\frac{\ln(5)}{2} = \frac{2x}{2}$ $\boxed{x = 0.804}$
40. $e^{2x} - 4e^x - 12 = 0$ $(e^x - 6)(e^x + 2) = 0$ $e^x = 6$ $e^x = -2$ $\ln(6) = \boxed{x = 1.791}$ $\cancel{\ln(-2)} = x$	41. $\frac{14e^{3x+2}}{14} = \frac{560}{14}$ $e^{3x+2} = 40$ $\ln(40) = 3x + 2$ $-2 \mid -2$ $\frac{\ln(40) - 2}{3} = \frac{3x}{3}$ $\boxed{x = 0.562}$	42. $\frac{5(10^{x-6})}{5} = \frac{7}{5}$ $10^{x-6} = 7/5$ $\log(7/5) = x - 6$ $+6$ $+6$ $x = \log(7/5) + 6$ $\boxed{x = 6.146}$	43. $\frac{8(10^{3x})}{8} = \frac{12}{8}$ $10^{3x} = 1.5$ $\frac{\log(1.5)}{3} = \frac{3x}{3}$ $\boxed{x = 0.058}$

You may use a calculator… $r = 0.043$

44. The population of Newmanville is growing at a rate of 4.3% per year. There are now 20,000 people in town.

a. Write a general equation to express the growth rate.

$$A = 20{,}000(1+0.043)^t$$

b. How many people will there be in town in 12 years?

$$20{,}000(1+0.043)^{12} = 33146.806 \approx \boxed{33{,}146 \text{ people}}$$

c. When will there be 100,000 people?

$$\frac{100{,}000}{20{,}000} = \frac{20{,}000(1+0.043)^t}{20{,}000}$$

$$1.043^t = 5$$

$$\log_{1.043}(5) = t$$

$$\boxed{t = 38.227}$$

45. Jordan knows that her 24 grams of Iodine-131 has a half-life of 8 days.

a. How much will be left after 22 days?

$$A = 24(0.5)^{22/8} = \boxed{3.567 \text{ g}}$$

b. When will there be 6 grams?

$$\frac{6}{24} = \frac{\cancel{24}(0.5)^{t/8}}{\cancel{24}} \rightarrow 0.5^{t/8} = 0.25$$

$$(8)\log_{0.5}(0.25) = \frac{t}{\cancel{8}}(\cancel{8})$$

$$\boxed{t = 16 \text{ days}}$$

45. Jordan knows that her 24 grams of Iodine-131 has a half-life of 8 days.

a. How much will be left after 22 days?

b. When will there be 6 grams?

46. What is the growth rate of 19 bacteria if after 3.5 hours there are 2000 bacteria? (continuous growth)

P t A

$$\frac{2000}{19} = \frac{\cancel{19}e^{r(3.5)}}{\cancel{19}}$$

$$e^{3.5r} = \frac{2000}{19}$$

$$\frac{\ln\left(\frac{2000}{19}\right)}{3.5} = \frac{\cancel{3.5}r}{\cancel{3.5}}$$

$$\boxed{r = 1.330 \text{ or } 133\%}$$

47. Suppose that $5000 is invested at 4% compounded quarterly. After t years, you have $7000. How many years was the money invested?

P $r = 0.04$ $n = 4$ A

$$\frac{7000}{5000} = \frac{\cancel{5000}\left(1 + \frac{0.04}{4}\right)^{4t}}{\cancel{5000}}$$

$$1.01^{4t} = 1.4$$

$$\frac{\log_{1.01}(1.4)}{4} = \frac{\cancel{4}t}{\cancel{4}}$$

$$\boxed{t = 8.453 \text{ years}}$$

48. You invest $2500 in an account at an interest rate r, compounded continuously.

P e

a) Find the time needed for the investment to double if invested at a rate of 8.5%.

$A = 5000$ $r = 0.085$

$$\frac{5000}{2500} = \frac{\cancel{2500}e^{0.085t}}{\cancel{2500}}$$

$$e^{0.085t} = 2$$

$$\frac{\ln(2)}{0.085} = \frac{\cancel{0.085}t}{\cancel{0.085}}$$

$$\boxed{t = 8.154 \text{ years}}$$

b) Find the time needed for the investment to triple if invested at a rate of 8.5%.

$A = 7500$ $r = 0.085$

$$\frac{7500}{2500} = \frac{\cancel{2500}e^{0.085t}}{\cancel{2500}}$$

$$e^{0.085t} = 3$$

$$\frac{\ln(3)}{0.085} = \frac{\cancel{0.085}t}{\cancel{0.085}}$$

$$\boxed{t = 12.924 \text{ years}}$$