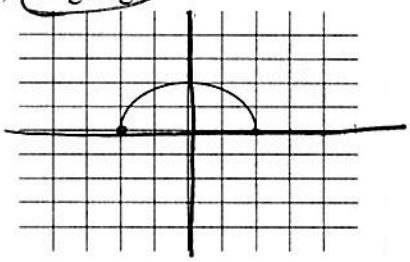
PAP PreCal Review - Parametrics & Polars Name KEY

1. Write the parametric equations that model the given function, using trig

$x = 2\cos T$

$y = 2\sin T$

$0 \le t \le \pi$ (for top ½)



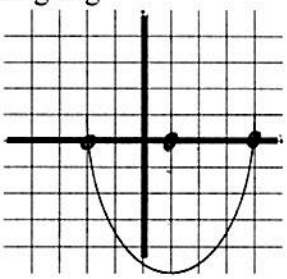
circle
center (0,0)
r = 2

2. Write the transformations of the above parametric equations, using trig

$x = 3\cos T + 1$

$y = 5\sin T$

$\pi \le t \le 2\pi$ (for bottom ½)



ellipse
center (1,0)
x rad = 3
y rad = 5

3) Gonzo is being shot out of a cannon. He will be shot with an initial velocity of 150 ft/sec at a 40° angle, and the when he exits the cannon he will be 3 ft off the ground. A mat is located 700 ft away from the cannon and is 10 feet long.

a) Write your parametric equations. $x = 150T\cos 40°$

$y = -16T^2 + 150T\sin 40° + 3$

b) Will he land on the mat, or will he go SPLAT?

mat: $x = 700$ $y = 10$

SPLAT!

c) When will he be at his highest point? What will the height be? What will be his horizontal distance at that time? $t = 3$ sec. $y = 148.254$ ft $x = 344.72$ ft

d) Where will he be at 1.3 seconds?

(149.378, 101.303)

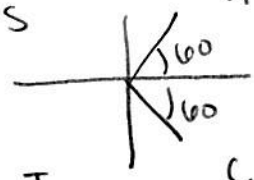
4) Find where the two polar graphs intersect. Give answers in radians.

$r = 4$; $r = 3 + 2\cos\theta$

$4 = 3 + 2\cos\theta$

$1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta$



$\pi/3$ A ✓

$5\pi/3$ ✓

$(4, \pi/3)$ & $(4, 5\pi/3)$

Identify the equation of the graph indicated. Circle the letter of your answer.

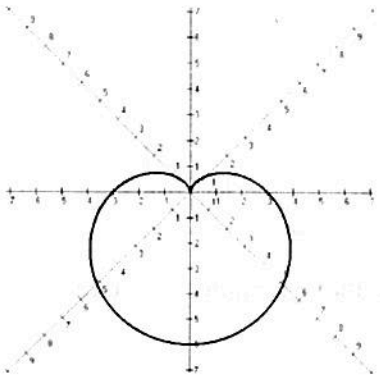
5.

A) $r = 3 - 3\sin\theta$

B) $r = 3 + 3\sin\theta$

C) $r = 3 - 3\cos\theta$

D) $r = 3 + 3\cos\theta$



cardioid

sin
(−)

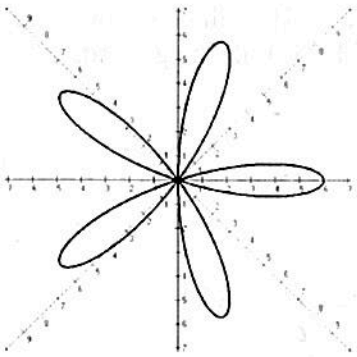
6.

E) $r = 6\sin(5\theta)$

F) $r = 6\cos(5\theta)$

G) $r = 6\sin(10\theta)$

H) $r = 6\cos(10\theta)$



rose
$n = 5$
$a = 6$
cos

7. Convert the following polar coordinates to rectangular. **Be able to do this without a calculator if the angles are on your unit circle. If they are not on the unit circle, then use a calculator.**

A) $\left(2, \frac{\pi}{4}\right)$ Q1

$x = 2\cos\frac{\pi}{4}$

$x = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$y = 2\sin\frac{\pi}{4}$

$y = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$

$(\sqrt{2}, \sqrt{2})$

B) $(-5, -60°)$ Q2

$x = -5\cos(-60°)$

$x = -5\left(\frac{1}{2}\right) = -\frac{5}{2}$

$y = -5\sin(-60°)$

$y = -5\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\sqrt{3}}{2}$

$\left(-\frac{5}{2}, \frac{5\sqrt{3}}{2}\right)$

C) $(-3, 193°)$ Q1

$x = -3\cos(193°)$

$x = 2.923$

$y = -3\sin(193°)$

$y = 0.674$

$(2.923, 0.674)$

D) $\left(6, \frac{6\pi}{7}\right)$ Q2

$x = 6\cos\frac{6\pi}{7}$

$x = -5.405$

$y = 6\sin\frac{6\pi}{7}$

$y = 2.603$

$(-5.405, 2.603)$

8. Convert the following rectangular coordinates to polar. **Be able to do this without a calculator if the angles are on your unit circle. If they are not on the unit circle, then use a calculator.**

A) $(\sqrt{3}, -1)$ Q4
x y

$r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4}$

$r = 2$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{11\pi}{6}$

$\left(2, \frac{11\pi}{6}\right)$

B) (0, 12)
x y

$r = \sqrt{0^2 + 12^2}$

$r = 12$

$\theta = \tan^{-1}\left(\frac{12}{0}\right)$ und

$\theta = \pi/2$

$(12, \pi/2)$

C) $(-\sqrt{5}, \sqrt{5})$ Q2
x y

$r = \sqrt{(-\sqrt{5})^2 + (\sqrt{5})^2}$

$r = \sqrt{10}$

$\theta = \tan^{-1}\left(\frac{\sqrt{5}}{-\sqrt{5}}\right)$

$\theta = 3\pi/4$

$(\sqrt{10}, 3\pi/4)$

D) $(-7\sqrt{3}, 7)$ Q2
x y
(radians)

$r = \sqrt{(-7\sqrt{3})^2 + 7^2}$

$r = 14$

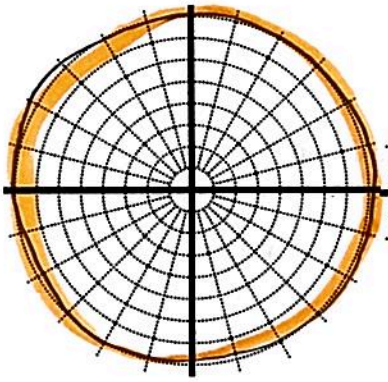
$\theta = \tan^{-1}\left(\frac{7}{-7\sqrt{3}}\right)$

$\theta = \frac{5\pi}{6}$

$\left(14, \frac{5\pi}{6}\right)$

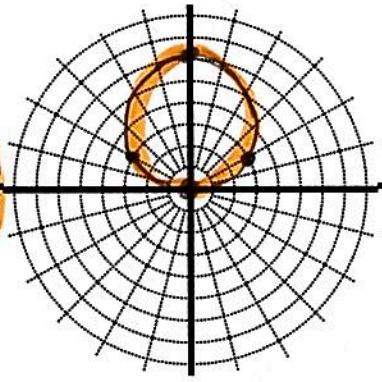
9. Graph each of the following on a polar grid.

A) $r = 8$



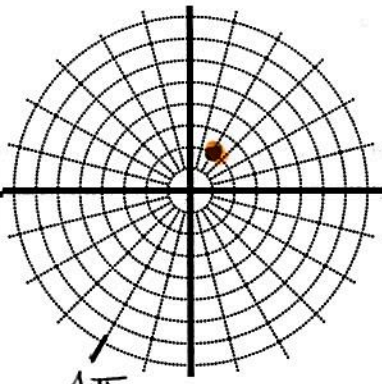
Circle

B) $r = 6 \sin\theta$



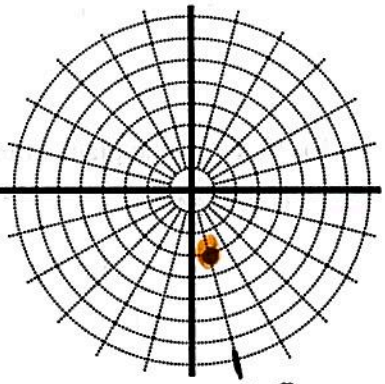
Circle

C) $(-2, \frac{4\pi}{3})$



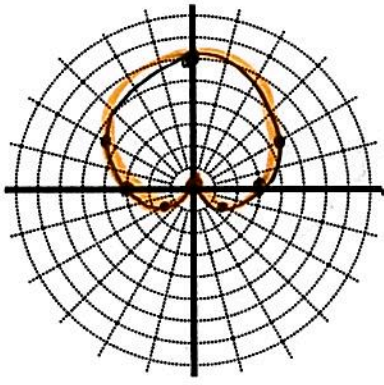
$\frac{4\pi}{3}$

D) $(3, -75^0)$



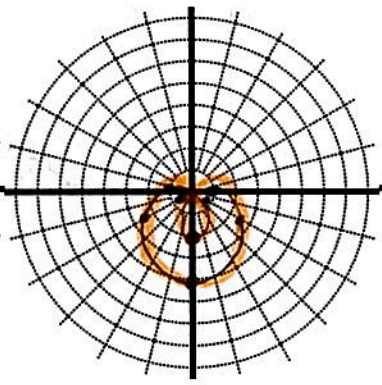
-75

E) $r = 3 + 3 \sin\theta$



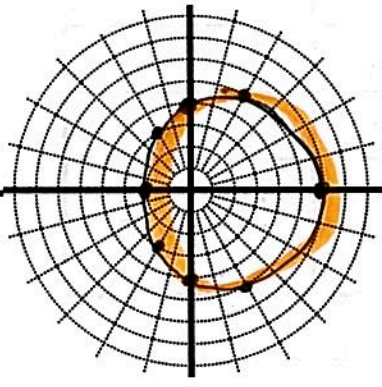
Cardioid

F) $r = 1 - 3 \sin\theta$



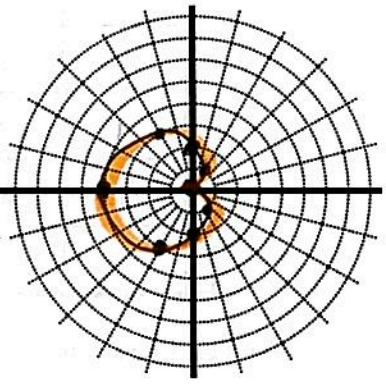
inner loop

G) $r = 4 + 2 \cos\theta$



Convex

H) $r = 2 - 2 \cos\theta$



Cardioid

Convert the parametric equations to rectangular equations by eliminating the parameter. (If it is a conic, you do not have to isoloate y, but if it is linear or quadratic please isolate y).

10. $x = 2t, \quad y = 4t$

$t = \frac{x}{2}$

$y = 4\left(\frac{x}{2}\right)$

$y = 2x$

11. $x = t + 4, y = t^2$

$t = x - 4$

$y = (x-4)^2$

or

$y = x^2 - 8x + 16$

12. $x = 5\cos t$ (a)

$x = 6\sin t$ (b)

ellipse

center $(0,0)$

$\frac{x^2}{25} + \frac{y^2}{36} = 1$

13. $x = 6\cos t$

$y = 6\sin t$

circle

center $(0,0)$

$r = 6$

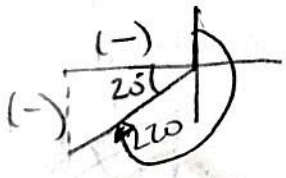
$x^2 + y^2 = 36$

14. A plane is flying on a heading of 250° at a speed of 220 mph. Where will the plane be in 5 hours?

a) Find equations for the plane

$x = -220T\cos 20°$

$y = -220T\sin 20°$

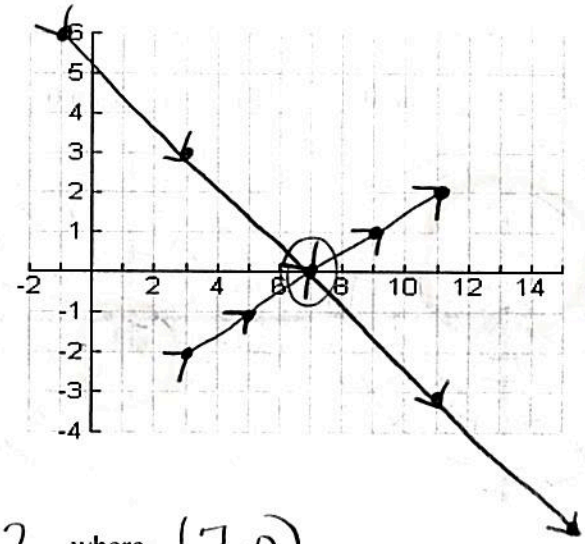


b) Where is the plane after 5 hours? (Your answer should have both a vertical and a horizontal component.)

$(-1033.661, -376.222)$ or 1033.661 mi West, 376.222 mi South

15. Graph the parametric equations $x = 3 + 2t$, $y = -2 + t$ and $x = -1 + 4t$, $y = 6 - 3t$ on $[0, 4]$.

t	x	y	x	y
0	3	−2	−1	6
1	5	−1	3	3
2	7	0	7	0
3	9	1	11	−3
4	11	2	15	−6



a) Do their lines of travel intersect? Yes

b) Do the two objects collide? Yes If so, when? $t = 2$ where $(7, 0)$.