re-AP Precalculus

3.4 Exponential Equations

olving Exponential Equations

.. If possible, rewrite the equation where both sides are expressed as powers of the same base. Then, the xponents will be equal to each other and you can solve.

:. If the two sides cannot be written as powers of the same base, then isolate the exponential portion, take he log or ln of both sides and solve for x. Note—you might need to use power rule and/or factoring as you re working.

)

1) $e^{-x^2} = e^{5x+6}$

$-x^2 = 5x+6$

$x^2+5x+6=0$

$(x+2)(x+3)=0$

$x=-2,-3$

2) $4^x = 32$

$(2^2)^x = 2^5$

$2^{2x} = 2^5 \rightarrow \frac{2x}{2} = \frac{5}{2}$

$x = 2.5$

3) $5-3e^x = 2$

$-5 \quad -5$

$\frac{-3e^x}{-3} = \frac{-3}{-3}$

$e^x = 1 \rightarrow \ln 1 = x$

$x = 0$

4) $6(2^{x+5}) + 4 = 11$

$-4 \quad -4$

$\frac{6(2^{x+5})}{6} = \frac{7}{6}$

$2^{x+5} = \frac{7}{6} \rightarrow \log_2\left(\frac{7}{6}\right) = x+5$

$-5 \quad -5$

$x = \log_2(7/6) - 5$ or -4.777

5) $e^{3x} = 5$

$\frac{\ln 5}{3} = \frac{3x}{3}$

$x = \frac{\ln 5}{3}$ or 0.536

6) $e^{3x} = 8$

$\frac{\ln 8}{3} = \frac{3x}{3}$

$x = \frac{\ln 8}{3}$ or 0.693

7) $e^{2x} - 7e^x + 12 = 0$

$(e^x-3)(e^x-4)=0$

$e^x = 3$ $\quad$ $e^x = 4$

$x = \ln 3$ or 1.098 $\quad$ $x = \ln 4$ or 1.386

8) $e^{2x} - 3e^x + 2 = 0$

$(e^x-2)(e^x-1)=0$

$e^x = 2$ $\quad$ $e^x = 1$

$x = \ln 2$ or 0.693 $\quad$ $x = \ln 1 = 0$

3.4 Logarithmic Equations

Solving Logarithmic Equations

1. If possible, rewrite the equation as two logs **(with the same base)** set equal to each other. You might need to use log properties to make that happen. Then set arguments equal to each other and solve.
2. If there are non-log terms, put all the log terms on one side and the non-log terms on the other. Then use log properties to write the log terms as a single log and convert to an exponential.
3. **Make sure to check all solutions**…if any make the original argument(s) negative then they are extraneous and should be excluded.

1) $\ln x = \ln 3 + 1/2$

$-\ln 3 \quad -\ln 3$

$\ln x - \ln 3 = \frac{1}{2}$

$\ln\left(\frac{x}{3}\right) = \frac{1}{2}$

(e) $e^{1/2} = \frac{x}{3}$ (3)

$x = 3e^{1/2}$ or 4.946

2) $\log_4(3x + 2) = \log_4(6 - x)$

$3x + 2 = 6 - x$

$+x \quad -2 \quad -2 \quad +x$

$\frac{4x}{4} = \frac{4}{4}$

$x = 1$

3) $\log_3(5x + 13) - \log_3 6 = \log_3 3x$

$\log_3\left(\frac{5x+13}{6}\right) = \log_3 3x$

$\frac{5x+13}{6} = \frac{3x}{1}$

$18x = 5x + 13$

$-5x \quad -5x$

$13x = 13 \rightarrow x = 1$

4) $6 + 3\ln x = 4$

$-6 \quad -6$

$\frac{3\ln x}{3} = \frac{-2}{3}$

$\ln x = -2/3$

(e) $x = e^{-2/3}$ or 0.513

5) $\frac{3}{3}\log_4 6x = \frac{9}{3}$

$\log_4(6x) = 3$

$4^3 = 6x$

$\frac{64}{6} = \frac{6x}{6}$

$x = \frac{32}{3}$

6) $\log_{10} x + \log_{10}(x - 9) = 1$

$\log(x(x-9)) = 1$ (10)

$10^1 = x^2 - 9x$

$x^2 - 9x - 10 = 0$

$(x-10)(x+1)$

$x = 10 \quad -1$ ← ext!

7) $\log_{\frac{1}{2}}\left(\frac{1}{32}\right) = x$

$8^x = \frac{1}{32}$

$\left(\frac{2}{3}\right)^x = 2^{-5}$

$x = -5/3$ or $-1.\overline{6}$

8) $\log_9(243) = x$

$9^x = 243$

$(3^2)^x = 3^5$

$\frac{2x}{2} = \frac{5}{2}$

$x = 2.5$

$\frac{3x}{3} = \frac{-5}{3}$