

3.4 Solving Exponential & Log Equations

Solve the following. Do not use a calculator (leave your answers exact).

1.  $(\frac{1}{8})^x = 64$

~~8~~<sup>-x</sup> = ~~8~~<sup>2</sup>

~~1~~<sup>x</sup> = ~~2~~<sub>-1</sub>     **X = -2**

2.  $\frac{6(10^x)}{6} = \frac{216}{6}$

$(10)^x = 36$

**$\log_{10} 36 = x$**

3.  $2^{x+3} = 256$

~~2~~<sup>x+3</sup> = ~~2~~<sup>8</sup>

~~x+3~~ = ~~8~~<sub>-3</sub>     **X = 5**

4.  $3^{x-1} = \frac{1}{81}$

~~3~~<sup>x-1</sup> = ~~3~~<sup>-4</sup>

~~x-1~~ = ~~-4~~<sub>+1</sub>     **X = -3**

5.  $\ln x - \ln 5 = 0$

$\ln(\frac{x}{5}) = 0$

X = 5(1)

(5)e<sup>0</sup> = ~~x~~<sub>5</sub>     **X = 5**

6.  $\ln(2x-1) = 5$

e<sup>5</sup> = 2x-1

~~e<sup>5</sup>~~ + 1 = ~~2x~~<sub>2</sub>

**$x = \frac{e^5 + 1}{2}$**

7.  $\log_3 x + \log_3 4 = \frac{1}{4} \log_3 81$

$\log_3(4x) = \log_3 \sqrt[4]{81}$

~~4x~~ = ~~3~~<sub>4</sub>

**X = 3/4**

8.  $-1 + 2 \ln 3x = 17$

~~2~~ ln 3x = ~~18~~<sub>2</sub>

$\ln 3^x = 9$

$\frac{e^9}{3} = 3x$

**$x = \frac{e^9}{3}$**

9.  $\log_4 x - \log_4(x-1) = \frac{1}{2}$

$\log_4(\frac{x}{x-1}) = \frac{1}{2}$

~~4~~<sup>1/2</sup> = ~~x~~<sub>x-1</sub>

~~2~~ = ~~x~~<sub>x-1</sub>     ~~2x-2~~ = ~~x~~<sub>-x+2</sub>     **X = 2**

Simplify the following expressions.

10.  $\ln e^x$

~~x~~<sup>2</sup> ~~ln e~~     **X<sup>2</sup>**

11.  $5 + e^{\ln(x^2+1)}$

5 + x<sup>2</sup> + 1 → **X<sup>2</sup> + 6**

12.  $\frac{1}{3} \log_3 5$

**$\frac{1}{3}$**

13.  $\log_2(\log_3 81)$

$\log_2(\log_3 3^4)$

$\log_2(4 \log_3 3)$

~~log<sub>2</sub> 2~~<sup>2</sup> → 2 ~~log<sub>2</sub> 2~~ = **2**

Solve the following. Use your calculator to check your answer. Round your result to three decimal places.

14.  $\frac{500e^{-x}}{500} = \frac{300}{500}$

$e^{-x} = \frac{3}{5}$   
 $\ln \frac{3}{5} = -x$

$x = -\ln \frac{3}{5}$  or 0.511

15.  $7 - 2e^x = 5$

$-2e^x = -2$

$e^x = 1$

$x = 0$

$\ln 1 = x$

16.  $5(2^{3-x}) - 13 = 100$

$5(2^{3-x}) = \frac{113}{5}$

$2^{3-x} = \frac{113}{25}$

$\log_2 \left( \frac{113}{25} \right) = 3 - x$

$-x = \log_2 \left( \frac{113}{25} \right) - 3$

$x = -\log_2 \left( \frac{113}{25} \right) + 3$   
 or -1.498

17.  $e^{2x} - 4e^x + 5 = 0$

$(e^x - 5)(e^x + 1) = 0$

$e^x = 5$

$e^x = -1$

$x = \ln 5 = 1.609$

$x = \ln(-1)$   
 extraneous!

18.  $e^{2x} - 5e^x + 6 = 0$

$(e^x - 2)(e^x - 3) = 0$

$e^x = 2$

$e^x = 3$

$x = \ln 2 = 0.693$

$x = \ln 3 = 1.099$

19.  $e^{x^2-3x} = e^{x-2}$

$x^2 - 3x = x - 2$

$x^2 - 4x + 2 = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{8}}{2}$

$\frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

20.  $\ln 4x = 2.1$

$\frac{e^{2.1}}{4} = x$

$x = \frac{e^{2.1}}{4}$  or 2.042

21.  $\ln(x+5) = \ln(x-1) - \ln(x+1)$

$\ln(x+5) = \ln \left( \frac{x-1}{x+1} \right)$

$\frac{x+5}{1} = \frac{x-1}{x+1}$

$x^2 + 6x + 5 = x - 1$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2, -3$

Both are extraneous solutions (neg. argument)  
 $\Rightarrow$  No solution