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Rational Functions Notes – Day 1

Asymptotes and Removable Discontinuities

A **rational function** is a function that can be written as a fraction of two polynomials:

$$f(x) = \frac{p(x)}{q(x)}$$

Key Features of Rational Function Graphs:

1. **Vertical Asymptotes**—imaginary vertical lines that the graph gets close to but never actually touches. They occur where $q(x) = 0$ {once the function has been reduced}

2. **Hole or Removable Discontinuity**—a hole or gap in the graph. This only happens when the numerator and denominator have a common factor (can be reduced). For instance, if (x – 2) is a factor in the top and bottom, there will be a hole at x = 2. To find the y-value of the hole's ordered pair, plug 2 in for x in the reduced equation.

3. **Horizontal asymptote**—imaginary horizontal line that the graph gets close to but never actually touches. (Technically a graph will sometimes cross a horizontal asymptote but we will look at that later.)

- If the highest power of x is in the denominator, then the horizontal asymptote is y = 0.

  EX. $\dfrac{x^2 - 3x + 2}{x^3 - 3}$

- If the highest power of x is the same in the numerator & denominator, then the horizontal asymptote will be the coefficients in front of those terms.

  EX. $\dfrac{2x^2 - 3x + 2}{3x^2 - 3}$

- If the highest power of x is in the numerator, then there is no horizontal asymptote

  EX. $\dfrac{x^3 - 3x + 2}{x^2 - 5}$

4. **Slant Asymptotes** - When the highest power of x is in the numerator (or the top of the equation), there is no horizontal asymptote. Instead, our asymptote is some other kind of function. If the degree on top is only one higher than the degree on the bottom, then the asymptote is a slanted line, or a slant asymptote. To find this, you use long division to divide the numerator (top) by the denominator (bottom). Disregard the remainder, the quotient is your equation for the slant asymptote.

1) $f(x) = \frac{x-2}{x+5}$

RD? None
VA? $X = -5$
HA? $Y = 1$ COCO $\frac{1}{1}$
SA? None

2) $f(x) = \frac{3x-9}{x^2-9} = \frac{3(\cancel{x-3})}{(x+3)(\cancel{x-3})} \rightarrow Y = \frac{3}{x+3}$ $\quad \frac{3}{3+3}$

RD? $(3, \frac{1}{2})$
VA? $X = -3$
HA? $Y = 0$ BOBO
SA? None

3) $f(x) = \frac{5}{x^2-3x-4} = \frac{5}{(x-4)(x+3)}$

RD? None
VA? $X = 4,\ X = -3$
HA? $Y = 0$ BOBO
SA? None

4) $f(x) = \frac{4x^2-24x+32}{x^2+x-20} = \frac{4(x^2-6x+8)}{} = \frac{4(\cancel{x-4})(x-2)}{(x+5)(\cancel{x-4})} \rightarrow \frac{4(x-2)}{x+5}$ $\quad \frac{4(4-2)}{4+5} = \frac{8}{9}$

RD? $(4, \frac{8}{9})$
VA? $X = -5$
HA? $Y = 4$ COCO $\frac{4}{1}$
SA? None

5) $f(x) = \frac{x^2-16}{x+4} = \frac{(\cancel{x+4})(x-4)}{\cancel{x+4}} \rightarrow Y = X - 4$ $\quad -4-4 = -8$

RD? $(-4, -8)$
VA? None
HA? None
SA? None

linear equation when simplified

6) $f(x) = \frac{x^2+x-2}{x^2-5x+4} = \frac{(x+2)(\cancel{x-1})}{(x-4)(\cancel{x-1})} \rightarrow \frac{x+2}{x-4}$ $\quad \frac{1+2}{1-4} = \frac{3}{-3}$

RD? $(1, -1)$
VA? $X = 4$
HA? $Y = 1$ COCO $\frac{1}{1}$
SA? None

7) $f(x) = \frac{3x^2-9x-30}{x^2-x-20} = \frac{3(x^2-3x-10)}{} = \frac{3(\cancel{x-5})(x+2)}{(\cancel{x-5})(x+4)}$ $\quad \frac{3(x+2)}{x+4}$ $\quad \frac{3(5+2)}{5+4} = \frac{21}{9}$

RD? $(5, \frac{21}{9})$
VA? $X = -4$
HA? $Y = 3$
SA? None

COCO $\frac{3}{1}$

8) $f(x) = \frac{2x^3-2x}{x^3-x^2-9x+9} = \frac{2x(x^2-1)}{x^2(x-1)-9(x-1)}$

RD? $(1, \frac{1}{2})$
VA? $X = -3,\ X = 3$
HA? $Y = 2$ COCO $\frac{2}{1}$
SA? None

$\frac{2x(x+1)(\cancel{x-1})}{(x+3)(x-3)(\cancel{x-1})}$ $\quad \frac{2(2)}{4(2)}$

9) $f(x) = \frac{4x^2-8x-12}{x^3-2x^2-9x+18} = \frac{4(x^2-2x-3)}{x^2(x-2)-9(x-2)}$

RD? $(3, \frac{8}{3})$
VA? $X = -3,\ X = 2$
HA? $Y = 0$ BOBO
SA? None

$\frac{4(\cancel{x-3})(x+1)}{(x+3)(\cancel{x-3})(x-2)}$

$\frac{4(x+1)}{(x+3)(x-2)}$ $\quad \frac{4(4)}{6(1)} = \frac{16}{6} = \frac{8}{3}$

10) $f(x) = \frac{x^2+2}{x+1}$

RD? None
VA? $X = -1$
HA? None BOTN
SA? $Y = X - 1$

$$\begin{array}{r} x-1 \\ x+1 \overline{) x^2 + 0x + 2} \\ -(x^2 + x) \downarrow \\ -x + 2 \\ -(-x - 1) \\ \hline 3\ R \end{array}$$