Projectile Motion Notes

Parametric Equations are often used to model projectile motion because projectile motion has both a horizontal and vertical component that are independent of each other.

Horizontal component: $x = v_0 T \cos\theta$

Vertical component: $y = -16T^2 + v_0 T \sin\theta + h$

(v_0 is the initial velocity of the object and $16t^2$ is the effect of gravity in ft^2/sec pulling the object downward.

EX1: Write a parametric equation for the path of a cannonball shot from a cannon, 2 feet off the ground, with an initial velocity of 87 ft^2/s and an angle of elevation of 60°.

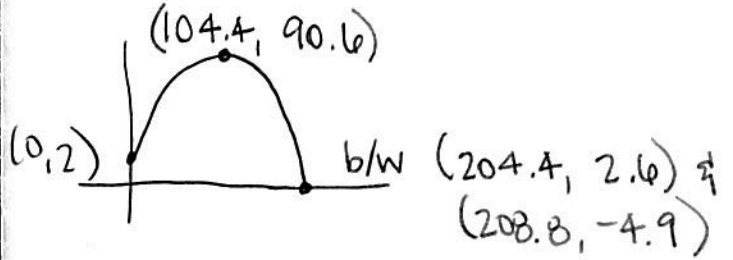
h, v_0, θ

x(t)= $87T\cos(60°)$

y(t)= $-16T^2 + 87T\sin(60°) + 2$

Calculator Window:
Tmin: 0 Tmax: 5 Tstep: .1
Xmin: 0 Xmax: 250 xscl: 1
Ymin:0 ymax:100 yscl:1

A. Sketch a graph of this equation in parametric mode. Label starting point, highest point, and end using TRACE.



B. Approximate when the cannonball hits the ground. Also practice checking the table to approximate answer.

b/w 4.7-4.8 sec.

C. If there was a 30 (y) ft wall 190 (x) feet from the cannon, would the cannonball clear the wall?

$X_2 = 190$
$Y_2 = 30$ →

$X_{min} = 180$ $Y_{min} = 25$
$X_{max} = 200$ $Y_{max} = 35$

NO!

D. What if there was a 35 (y) ft wall 180 (x) feet from the cannon?

$X_2 = 180$
$Y_2 = 35$ →

$X_{min} = 170$ $Y_{min} = 30$
$X_{max} = 190$ $Y_{max} = 40$

Yes!

EX2: A soccer ball is kicked when the ball is 1 (h) feet above the ground. It leaves the foot with an initial velocity of 90 (V_0) ft^2/sec and at an angle of elevation of 30° (θ). The goal is 200 feet from the player and the top of the goal is 8 ft. off the ground.

Write a parametric equation to represent the path of the ball.

$X(t) = 90T\cos(30°)$

$Y(t) = -16T^2 + 90T\sin(30°) + 1$

$X_{min} = 0$
$X_{max} = 250$
$Y_{min} = 0$
$Y_{max} = 50$

When did the ball reach its maximum height?

$t =$ 1.4 sec. @ (109.1, 32.6)

Did he score a goal? Does the ball need to go **OVER** or **UNDER**? Justify your answer.

$X_2 = 200$
$Y_2 = 8$ → No, ball went over goal!

EX3: A baseball is hit at a point 3 (h) feet above the ground toward the left field fence. The fence is 10 feet high and 375 feet from home plate. If he hits the ball at an angle 30° (θ) with an initial velocity of 120 (V_0) ft^2/sec will he hit a homerun?

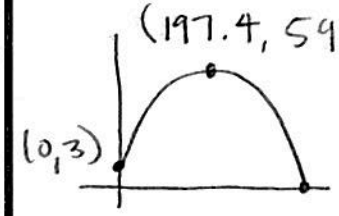
$x(t) = 120T\cos(30°)$

$y(t) = -16T^2 + 120T\sin(30°) + 3$

How do I need to change my window?

$X_{min} = 0$ $Y_{min} = 0$
$X_{max} = 400$ $Y_{max} = 100$

Sketch a graph of this equation in parametric mode. Label the starting point, the highest point, and <u>where</u> the ball lands.



b/w (384.5, 5.9) & (394.9, -0.04)

Does he hit a homerun?

Want over fence!

$X_2 = 375$
$Y_2 = 10$

$X_{min} = 365$ $X_{max} = 385$
$Y_{min} = 5$ $Y_{max} = 15$

Yes!