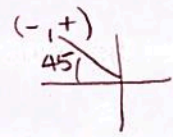


Sum and Difference and Double Angle Practice

For #1-2, find the exact value of each expression.



$$1. \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) \rightarrow \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$2. \sin(135^\circ - 30^\circ)$$

$$\sin(135)\cos(30^\circ) - \cos(135^\circ)\sin(30^\circ)$$

$$\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \rightarrow \frac{\sqrt{6} + \sqrt{2}}{4}$$

3. Find the exact values of the sine, cosine, and tangent of the angle: $195^\circ = 225^\circ - 30^\circ$

$$\sin(225^\circ - 30^\circ)$$

$$\sin(225)\cos(30) - \cos(225)\sin(30)$$

$$-\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \rightarrow \frac{-\sqrt{6} + \sqrt{2}}{4}$$

$$\cos(225^\circ - 30^\circ)$$

$$\cos(225)\cos(30) + \sin(225)\sin(30)$$

$$-\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow \frac{-\sqrt{6} - \sqrt{2}}{4}$$

$$\tan(225^\circ - 30^\circ)$$

$$\frac{\tan(225) - \tan(30)}{1 + \tan(225)\tan(30)}$$

$$\frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\left(\frac{\sqrt{3}}{3}\right)} \rightarrow \frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}}$$

For #4-5, write the expression as the sine, cosine, or tangent of an angle.

$$\frac{3 - \sqrt{3}}{3} \cdot \frac{3}{3 + \sqrt{3}} \rightarrow \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

4. $\cos 60^\circ \cos 10^\circ - \sin 60^\circ \sin 10^\circ$

$$\cos(60 + 10)$$

$$\cos(70^\circ)$$

5. $\frac{\tan 325^\circ - \tan 116^\circ}{1 + \tan 325^\circ \tan 116^\circ}$

$$\tan(325 - 116)$$

$$\tan(209^\circ)$$

For #6-8, find the exact value of the expression.

6. $\sin \frac{\pi}{12} \cos \frac{\pi}{4} + \cos \frac{\pi}{12} \sin \frac{\pi}{4}$

$$\sin\left(\frac{\pi}{12} + \frac{\pi}{4}\right)$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

7. $\sin 120^\circ \cos 60^\circ - \cos 120^\circ \sin 60^\circ$

$$\sin(120 - 60)$$

$$\sin(60) = \frac{\sqrt{3}}{2}$$

8. $\cos 120^\circ \cos 30^\circ + \sin 120^\circ \sin 30^\circ$

$$\cos(120 - 30)$$

$$\cos(90) = 0$$

9. Find the solution(s) of the equation in the interval $[0, 2\pi)$.

$$\cos\left(x + \frac{\pi}{4}\right) + \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\cancel{\cos x \cos\left(\frac{\pi}{4}\right)} - \cancel{\sin x \sin\left(\frac{\pi}{4}\right)} + \cos x \cos\left(\frac{\pi}{4}\right) + \cancel{\sin x \sin\left(\frac{\pi}{4}\right)} = 1$$

$$2 \cos x \cos\left(\frac{\pi}{4}\right) = 1$$

$$\cos x \left(\frac{\sqrt{2}}{2}\right) = \frac{1}{2} \rightarrow \frac{\sqrt{2}}{2} \cos x = \frac{1}{2} \rightarrow \cos x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{\pi}{4}, \frac{7\pi}{4}$$

Simplify the given expressions.

10. $2 \sin(10^\circ) \cos(10^\circ)$

$$\sin(2u) \rightarrow \sin(20^\circ)$$

12. $2 \sin(3x) \cos(3x)$

$$\sin(2u) \rightarrow \sin(6x)$$

14. $10 \sin(x) \cos(x)$

$$5(2 \sin x \cos x) \rightarrow 5 \sin(2x)$$

16. $\cos^2(5\theta) - \sin^2(5\theta)$

$$\cos(2u) \rightarrow \cos(10\theta)$$

11. $\cos^2(15^\circ) - \sin^2(15^\circ)$

$$\cos(2u) \rightarrow \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

13. $2 \cos^2 \theta - 2 \sin^2 \theta$

$$2(\cos^2 \theta - \sin^2 \theta) \rightarrow 2 \cos(2\theta)$$

15. $4 \sin(2x) \cos(2x)$

$$2(2 \sin(2x) \cos(2x)) \rightarrow 2 \sin(4x)$$

17. $\frac{\sin 2x}{2 \sin x}$

$$\frac{\cancel{2} \sin x \cos x}{\cancel{2} \sin x} \rightarrow \cos x$$

Solve 18-20 on the interval $0 \leq x < 2\pi$

* You can't divide by trig functions!

18. $\sin 2x \sin x = \cos x$

$$(2 \sin x \cos x)(\sin x) = \cos x$$

$$2 \sin^2 x \cos x - \cos x = 0$$

$$\cos x (2 \sin^2 x - 1) = 0$$

$$\cos x = 0$$

$$\sin x = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \text{ and } x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

19. $2 \sin(x) \cos(x) = \sin(x)$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x (2 \cos x - 1) = 0$$

$$\sin x = 0$$

$$\cos x = \frac{1}{2}$$

$$x = 0, \pi \text{ and } x = \frac{\pi}{3}, \frac{5\pi}{3}$$

20. $\sin(2x) + 2 \cos(x) = 0$

$$2 \sin x \cos x + 2 \cos x = 0$$

$$2 \cos x (\sin x + 1) = 0$$

$$\downarrow$$

$$2 \cos x = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x + 1 = 0$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$

21. Find all solutions of the equation in the interval $[0, 2\pi)$ if $\sin x \cos x = -\frac{1}{2}$ ⁽²⁾ $\rightarrow 2 \sin x \cos x = -1$

a. $\frac{3\pi}{2}, \frac{7\pi}{2}$

b. $\frac{5\pi}{6}, \frac{11\pi}{6}, \frac{2\pi}{3}, \frac{4\pi}{3}$

c. $\frac{3\pi}{4}, \frac{7\pi}{4}$

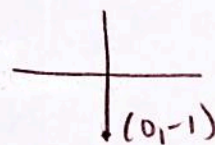
d. $\frac{3\pi}{2}$

$$\sin(2x) = -1$$

$$\frac{2x}{2} = \frac{3\pi}{2}$$

add new period $\frac{\pi}{1}$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$



22. True or False. Circle your answer.

a. $\tan 110^\circ = \frac{\tan 211^\circ - \tan 101^\circ}{1 + \tan 211^\circ \tan 101^\circ}$

$$= \tan(211 - 101)$$

$$= \tan(110^\circ)$$

True or False

b. $\cos 2(30^\circ) = 1$

$$2 \cos^2(30^\circ) - 1$$

$$2 \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$2 \left(\frac{3}{4}\right) - 1 = \frac{1}{2}$$

True or False

$$c. \sin(35^\circ + 20^\circ) \stackrel{u}{=} \sin 35^\circ + \stackrel{v}{\sin} 20^\circ$$

↓

$$\sin 35^\circ \cos 20^\circ + \cos 35^\circ \sin 20^\circ$$

True or **False**

$$d. \cos(45^\circ - 25^\circ) \stackrel{u}{=} \cos 45^\circ \cos 25^\circ - \sin 45^\circ \sin 25^\circ \quad \text{True or } \stackrel{v}{\text{False}}$$

$$\cos 45^\circ \cos 25^\circ + \sin 45^\circ \sin 25^\circ$$

True or **False**

$$e. 4 \sin 2x \cos 2x = 2 \sin 4x$$

$$2 \left(\underset{u}{2 \sin(2x)} \underset{u}{\cos(2x)} \right)$$

$$2 \sin(2 \cdot 2x)$$

$$2 \sin(4x)$$

True or **False**