Pre-AP Precalculus

Name key

Review for Test 2.2 – 2.5

Describe the right-hand and left-hand behavior of the graph of the polynomial function.

1) $f(x) = -x^2 + 6x + 9$

(-) even $\boxed{\downarrow\downarrow}$

2) $g(x) = -x^5 - 7x^2 + 10x$

(-) odd $\boxed{\uparrow\downarrow}$

Find all real zeros of the polynomial function.

3) $f(x) = 2x^2 + 11x - 21$

$(x+7)(2x-3)$

	$2x$	-3
x	$2x^2$	$-3x$
7	$14x$	-21

$\cdot\ -42$

$+\ 11$

$14\quad -3$

$\boxed{x = -7, 3/2}$

4) $f(t) = t^3 - 3t$

$t(t^2-3)$

$\boxed{t=0}$ $\quad \sqrt{t^2} = \pm\sqrt{3}$ $\quad \boxed{t = \pm\sqrt{3}}$

5) $f(x) = -12x^3 + 20x^2$

$-4x^2(3x-5)$

$\boxed{x=0}$ $\boxed{x = 5/3}$

6) $g(x) = x^4 - x^3 - 2x^2$

$x^2(x^2-x-2)$

$x^2(x-2)(x+1)$

$\boxed{x = 0, 2, -1}$

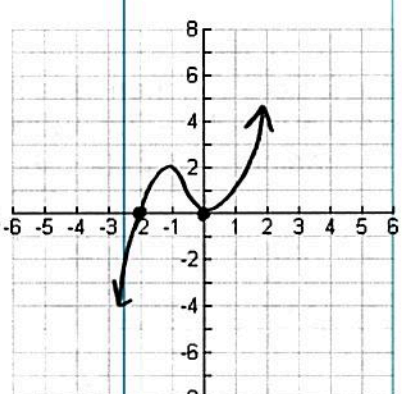
Sketch the graph of the function by applying the Leading Coefficient Test, finding the zeros of the polynomial, plotting sufficient solution points, and drawing a continuous curve through the points.

7) $f(x) = 2x^3 + 4x^2$ (+) odd $\downarrow\uparrow$

$2x^2(x+2)$

$x=0$ m2 bounce

$x=-2$ m1 cross

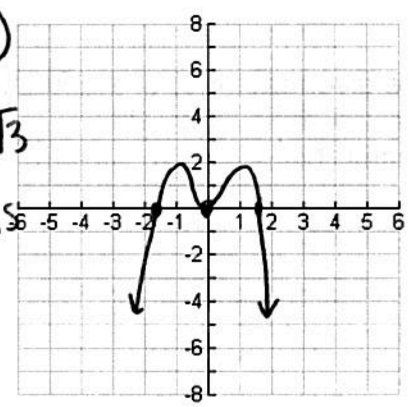


8) $h(x) = 3x^2 - x^4 \rightarrow -x^4 + 3x^2$ (-) even $\downarrow\downarrow$

$-x^2(x^2-3)$

$x=0$ m2 bounce

$x = \pm\sqrt{3}$ m1 cross



$+\sqrt{3} \approx 1.732$

$-\sqrt{3} \approx -1.732$

Use long division to divide.

9) $\dfrac{24x^2-x-8}{3x-2}$

$\boxed{8x+5+\dfrac{2}{3x-2}}$

$$\begin{array}{r} 8x+5 \\ 3x-2\,\overline{)\,\cancel{24}x^2-x-8} \\ \ominus\ \cancel{24}x^2-16x\ \downarrow \\ \hline \cancel{15}x-8 \\ \ominus\ \cancel{15}x-10 \\ \hline 2 \end{array}$$

10) $\dfrac{5x^3-13x^2-x+2}{x^2-3x+1}$

$\boxed{5x+2}$

$$\begin{array}{r} 5x+2 \\ x^2-3x+1\,\overline{)\,\cancel{5x}^3-13x^2-x+2} \\ \ominus\ \cancel{5x}^3-15x^2+5x\ \downarrow \\ \hline 2x^2-6x+2 \\ \ominus\ 2x^2-6x+2 \\ \hline 0 \end{array}$$

Use synthetic division to divide.

11) $\dfrac{6x^4-4x^3-27x^2+18x+0}{x-2}$ root $=2$

$$\begin{array}{r|rrrrr} 2 & 6 & -4 & -27 & 18 & 0 \\ \oplus & \downarrow & 12 & 16 & -22 & -8 \\ \hline & 6 & 8 & -11 & -4 & \lfloor -8\ R \\ & x^3 & x^2 & x & & \end{array}$$

$\boxed{6x^3+8x^2-11x-4+\dfrac{-8}{x-2}}$

12) $\dfrac{2x^3-19x^2+38x+24}{x-4}$ root $=4$

$$\begin{array}{r|rrrr} 4 & 2 & -19 & 38 & 24 \\ \oplus & \downarrow & 8 & -44 & -24 \\ \hline & 2 & -11 & -6 & \lfloor 0\ R \\ & x^2 & x & & \end{array}$$

$\boxed{2x^2-11x-6}$

Use synthetic division to determine whether the given values of x are zeros of the function.

13) f(x) = 20x⁴ + 9x³ – 14x² – 3x +0 a) x = -1 b) x = 0 c) ~~x = 1~~ d) $x = \dfrac{3}{4}$

$$\begin{array}{r|rrrrr} -1 & 20 & 9 & -14 & -3 & 0 \\ \oplus & \downarrow & -20 & 11 & 3 & 0 \\ \hline & 20 & -11 & -3 & 0 & \lfloor 0\ \checkmark \end{array}$$

$$\begin{array}{r|rrrrr} 0 & 20 & 9 & -14 & -3 & 0 \\ & \downarrow & 0 & 0 & 0 & 0 \\ \hline & 20 & 9 & -14 & -3 & \lfloor 0\ \checkmark \end{array}$$

$$\begin{array}{r|rrrrr} \cancel{1} & \cancel{20} & 9 & \cancel{-14} & -3 & 0 \\ & \downarrow & \cancel{20} & 29 & 15 & 12 \\ \hline & 20 & 21 & 15 & 12 & \lfloor \cancel{12} \end{array}$$

$$\begin{array}{r|rrrrr} \frac{3}{4} & 20 & 9 & -14 & -3 & 0 \\ & \downarrow & 15 & 18 & 3 & 0 \\ \hline & 20 & 24 & 4 & 0 & \lfloor 0\ \checkmark \end{array}$$

Verify the given factor(s) of the function f, find the remaining factors of f, then list all real zeros of f.

14) f(x) = x³ + 4x² – 25x – 28

factor: (x – 4), root = 4 → x = 4

$$\begin{array}{r|rrrr} 4 & 1 & 4 & -25 & -28 \\ \oplus & \downarrow & 4 & 32 & 28 \\ \hline & 1 & 8 & 7 & \lfloor 0\ R \\ & x^2 & x & & \end{array}$$

x^2+8x+7

$(x+7)(x+1)$

$x=-7$ $x=-1$

$\boxed{x=4,-7,-1}$

15) f(x) = x⁴ – 4x³ – 7x² + 22x + 24

factors: (x + 2) (x – 3)

x = -2 x = 3

$$\begin{array}{r|rrrrr} -2 & 1 & -4 & -7 & 22 & 24 \\ & \downarrow & -2 & 12 & -10 & -24 \\ \hline 3 & 1 & -6 & 5 & 12 & \lfloor 0 \\ & \downarrow & 3 & -9 & -12 & \\ \hline & 1 & -3 & -4 & \lfloor 0\ R & \\ & x^2 & x & & & \end{array}$$

$x^2-3x-4 \rightarrow (x-4)(x+1)$

$x=4$ $x=-1$

$\boxed{x=-2,3,4,-1}$

Find all the zeros of the function.

16) $f(x) = 3x(x-2)^2$

$\boxed{x=0}$ $\boxed{x=2}$

17) $f(x) = x^2 - 9x + 8$

$(x-8)(x-1)$

$\boxed{x = 8, 1}$

18) $f(x) = x^3 + 6x$

$x(x^2+6)$

$\sqrt{x^2} = \pm\sqrt{-6}$ < $\sqrt{-1}\,i$, $\sqrt{6}$

$\boxed{x=0}$ $\boxed{x = \pm i\sqrt{6}}$

19) $f(x) = (x+4)(x-6)(x-2i)(x+2i)$

$\boxed{x = -4, 6, \pm 2i}$

Find all zeros of the function and write the polynomial as a product of linear factors. (Factored form)

20) $f(x) = x^3 + 4x^2 - 5x$

$x(x^2+4x-5)$

$x(x+5)(x-1)$

Zeros: $x = 0, -5, 1$

Factored Form

$f(x) = x(x+5)(x-1)$

21) $g(x) = x^3 - 7x^2 + 36$ (+0x)

Possible Roots

$\pm 1, 2, 3, 4, 6, 9, 12, 18, 36$

3 | 1 -7 0 36
↓ 3 -12 -36
1 -4 -12 | 0 ✓
x^2 x

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6, -2$

Zeros: $x = 3, 6, -2$

Factored Form:

$f(x) = (x-3)(x-6)(x+2)$

22) $g(x) = x^3 + 6x^2 + 5x - 12$ ±1 ; ±1, 2, 3, 4, 6, 12

Possible Roots

$\pm 1, 2, 3, 4, 6, 12$

1 | 1 6 5 -12
↓ 1 7 12
1 7 12 | 0 ✓
x^2 x

$x^2 + 7x + 12$

$(x+3)(x+4)$

$x = -3, -4$

Zeros: $x = 1, -3, -4$

Factored Form

$f(x) = (x-1)(x+3)(x+4)$

23) $f(x) = 1x^4 + 6x^3 + 8x^2 - 6x - 9$ ±1 ; ±1, 3, 9

Possible Roots

$\pm 1, 3, 9$

1 | 1 6 8 -6 -9
↓ 1 7 15 9
1 7 15 9 | 0 ✓

-1 | 1 7 15 9
↓ -1 -6 -9
1 6 9 | 0 ✓
x^2 x

$x^2 + 6x + 9$

$(x+3)(x+3)$

$x = -3$

Zeros: $x = 1, -1, -3$

Factored Form:

$f(x) = (x-1)(x+1)(x+3)^2$

Perform the indicated operation. Write your answer in simplest form.

24) $(7 - 4i) + (-4 + 6i)$

$\boxed{3+2i}$

25) $(3 + 6i)^2$

$(3+6i)(3+6i)$

$9+18i+18i+36i^2(-1)$

-36

$\boxed{-27+36i}$

26) $\frac{3}{2+i} + \frac{7}{2-i}$

$\frac{3}{2+i}\cdot\frac{(2-i)}{(2-i)} + \frac{7}{2-i}\cdot\frac{(2+i)}{(2+i)} \rightarrow \underbrace{4-i^2(-1)}_{5}$

$\frac{6-3i+14+7i}{5} = \boxed{\frac{20+4i}{5}}$

27) Simplify $\frac{5}{-3i-5}$

$\frac{5}{-5+3i}\cdot\frac{(-5-3i)}{(-5-3i)} = \frac{-25-15i}{25-9i^2(-1)}$

$+9$

$\boxed{\frac{-25-15i}{34}}$

28) Simplify $i^{15} + i^{34} - i^{41} - i^{84}$

$i(i^2)^7 + (i^2)^{17} - i(i^2)^{20} - (i^2)^{42}$

$i(-1)^7 + (-1)^{17} - i(-1)^{20} - (-1)^{42}$

$i(-1) - 1 - i(1) - 1$

$-i - 1 - i - 1 = \boxed{-2-2i}$

29) Multiply $(\sqrt{7} + i\sqrt{34})$ by its conjugate

$(\sqrt{7}+i\sqrt{34})(\sqrt{7}-i\sqrt{34})$

$7 \cancel{- i\sqrt{238} + i\sqrt{238}} - i^2(34)$

$7 + (+1)(34) = \boxed{41}$

30) Find a polynomial function of degree 4 that has zeros at 4, -5, and 6i in both factored and standard form.

$\rightarrow (-6i)$ is also a zero

Factored Form: $(x-4)(x+5)(x-6i)(x+6i)$

$x^2+x-20 \qquad x^2+36$

	x^2	x	-20
x^2	x^4	x^3	$-20x^2$
36	$36x^2$	$36x$	-720

Standard Form: $x^4+x^3+16x^2+36x-720$