

Notes Sum and Difference Formulas

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Use sum/difference formulas to find exact values.

Examples:

Use Q1 & S $\rightarrow 30^\circ, 45^\circ, 60^\circ$

1. Find the exact value of $\cos 75^\circ$

$$\cos(30^\circ + 45^\circ) \rightarrow \cos(u + v)$$

$$\cos(30^\circ)\cos(45^\circ) - \sin(30^\circ)\sin(45^\circ)$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \rightarrow$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\text{or } \frac{\sqrt{6} - \sqrt{2}}{4}$$

2. Find the exact value of $\sin \frac{\pi}{12}$. (Hint: $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$)

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow$$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

Evaluating a Trigonometric Expression.

3. Find the exact value of $\sin 42^\circ \cos 12^\circ - \cos 42^\circ \sin 12^\circ$

$$\underbrace{\sin 42^\circ \cos 12^\circ - \cos 42^\circ \sin 12^\circ}_{\sin(u-v)}$$

$$\sin(42 - 12) \rightarrow \sin(30) = \boxed{\frac{1}{2}}$$

4. Find the exact value of $\cos \frac{5\pi}{36} \cos \frac{\pi}{9} - \sin \frac{5\pi}{36} \sin \frac{\pi}{9}$

$$\underbrace{\cos \frac{5\pi}{36} \cos \frac{\pi}{9} - \sin \frac{5\pi}{36} \sin \frac{\pi}{9}}_{\cos(u+v)}$$

$$\cos\left(\frac{5\pi}{36} + \frac{\pi}{9}\right)$$

$$\cos\left(\frac{5\pi}{36} + \frac{4\pi}{36}\right)$$

$$\cos\left(\frac{9\pi}{36}\right) \rightarrow \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$$

5. Find the exact value of $\frac{\tan(30^\circ) - \tan(60^\circ)}{1 + \tan(30^\circ)\tan(60^\circ)}$

$$\underbrace{\frac{\tan(30^\circ) - \tan(60^\circ)}{1 + \tan(30^\circ)\tan(60^\circ)}}_{\tan(u-v)}$$

$$\tan(30 - 60)$$

$$\tan(-30^\circ) = \boxed{-\frac{1}{\sqrt{3}} \text{ or } -\frac{\sqrt{3}}{3}}$$



$$u = x$$

$$v = \frac{\pi}{4}$$

For solving trigonometric equations.

6. $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = -1$ in the interval $[0, 2\pi)$

$$\sin(u+v) + \sin(u-v)$$

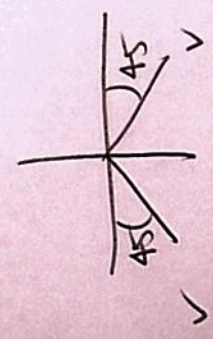
$$\sin(x)\cos\left(\frac{\pi}{4}\right) + \cos(x)\sin\left(\frac{\pi}{4}\right) + \sin(x)\cos\left(\frac{\pi}{4}\right) - \cos(x)\sin\left(\frac{\pi}{4}\right)$$

$$2\sin(x)\cos\left(\frac{\pi}{4}\right) = -1$$

$$\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\sqrt{2}}{\sqrt{2}} \sin(x) = -\frac{1}{\sqrt{2}}$$

$$\sin x = -\frac{1}{\sqrt{2}} \rightarrow -\frac{\sqrt{2}}{2}$$



$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$