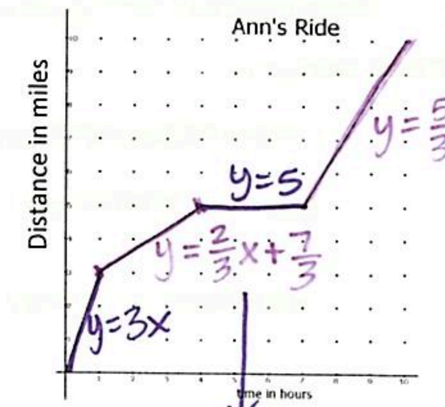
$y = \frac{5}{3}x + b$
$5 = \frac{5}{3}(7) + b$
$b = \frac{-20}{3}$

Lessson 1: Piecewise Functions

A piecewise function has different rules for different parts of its domain.

1. Ann went on a bicycle trip. The graph shows the relationship between time and distance traveled by Ann.



a. Write a function *d(t)* for her distance in miles traveled in terms of the time *(t)* in hours.

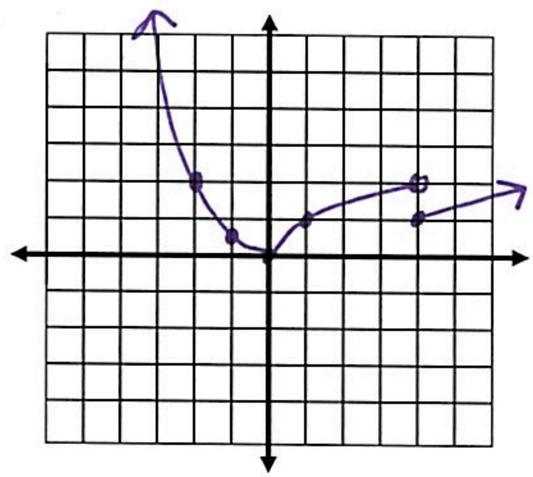
$$d(t) = \begin{cases} 3x, & 0 \le t \le 1 \\ \frac{2}{3}x + \frac{7}{3}, & 1 < t \le 4 \\ 5, & 4 < t \le 7 \\ \frac{5}{3}x - \frac{20}{3}, & 7 < t \le 10 \end{cases}$$

$y = \frac{2}{3}x + b$
$3 = \frac{2}{3}(1) + b$ $b = \frac{7}{3}$

b. Find *d(3)*. $\frac{2}{3}(3) + \frac{7}{3} = \boxed{\frac{13}{3}}$

2. Graph the following piece-wise function and state its domain and range.

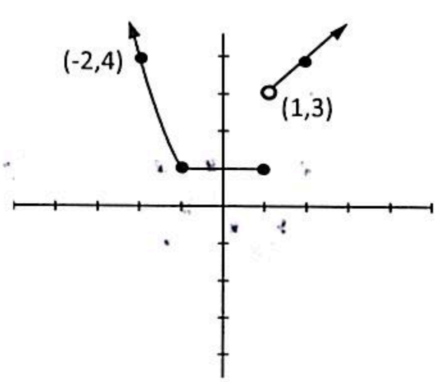
$$f(x) = \begin{cases} \frac{1}{2}x^2 & x \le 0 \\ \sqrt{x} & 0 < x < 4. \\ \frac{x}{4} & x \ge 4 \end{cases}$$



Evaluate: *f(0)=* 0 *f(4)=* 1 *f(1)=* 1

D: $(-\infty, \infty)$ R: $[0, \infty)$

3. Write the piece-wise function for the graph shown.



$$f(x) = \begin{cases} x^2, & x \le -1 \\ 1, & -1 < x \le 1 \\ x+2, & x > 1 \end{cases}$$

Continuity of Functions

Continuous - The function is a single unbroken curve. You can trace the function from left to right without lifting your pencil.

Discontinuous - This is the result if there is one or more discontinuity.

- Removable Discontinuity (hole) - the function would be continuous in the absence of the hole. if you "plug" the hole, the function becomes continuous.
- Jump Discontinuity - At a certain domain value, the function's value "jumps" and then continues. The jump creates a break in the curve where you must lift your pencil to trace it.
- Infinite Discontinuity - A vertical asymptote interrupts the continuity of the function.

In order for a function to be continuous at a point,

a) f(a) must exist

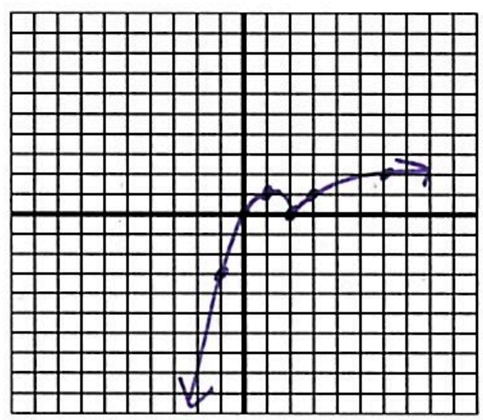
b) $\lim_{x \to a} f(x)$ must exist

c) f(a)= $\lim_{x \to a} f(x)$

discuss this more next class!

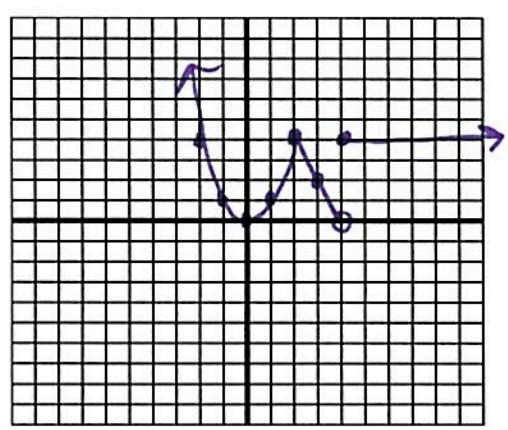
Sketch the graph of each function, then identify the types of discontinuities (if any).

4. $f(x) = \begin{cases} 1-(x-1)^2, & x \le 2 \\ \sqrt{x-2}, & x > 2 \end{cases}$



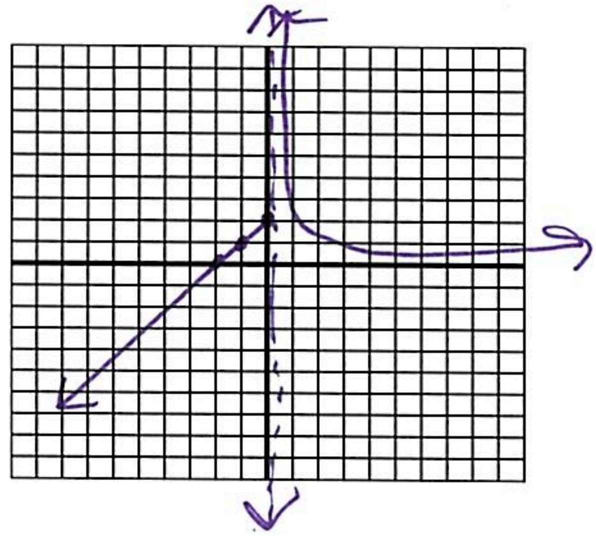
Discontinuities: none

5. $f(x) = \begin{cases} x^2, & x \le 2 \\ 8-2x, & 2 < x < 4 \\ 4, & x \ge 4 \end{cases}$



Discontinuities: jump @ x=4

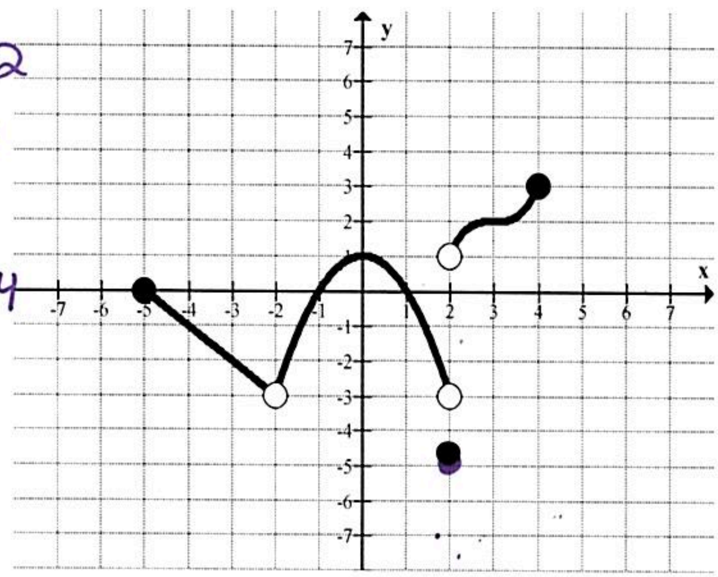
6. $f(x) = \begin{cases} x+2, & x \le 0 \\ \frac{1}{x}, & x > 0 \end{cases}$



Discontinuities: Infinite @ x=0 (vertical asymptote)

7. Write a piecewise defined function for the following graph.

$f(x) = \begin{cases} -x-5, & -5 \le x < -2 \\ -x^2+1, & -2 < x < 2 \\ -5, & x = 2 \\ (x-3)^3+2, & 2 < x \le 4 \end{cases}$



Find the following values:

f(-4) = -1

f(2) = -5

f(0) = 1

f(3) = 2

What is the absolute maximum? 3 The absolute minimum? -5

Find the interval(s) on which the function is increasing. $(-2,0)$ & $(2,4)$

Find the interval(s) on which the function is decreasing. $(-5,-2)$ & $(0,2)$

Where is this function discontinuous within the domain [-5,4]? $x=2$

What is the rate of change on the interval (-5, -2)? -1

Where is the rate of change zero? $x=0$, $x=3$

(levels off)