

Hyperbolas

Horizontal Hyperbola:

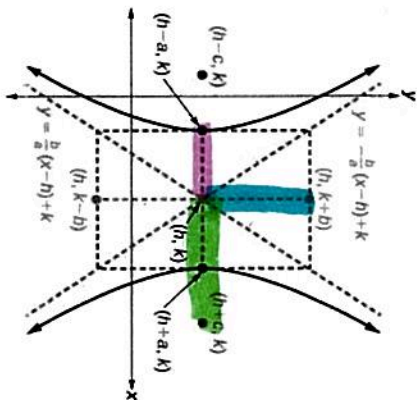
$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

1st denom

Conjugate axis connects the 2 covertices = 2b
Transverse axis connects the 2 vertices = 2a

Equation to find foci: $c^2 = a^2 + b^2$

slope of asymptotes = $\pm \frac{b}{a}$



Vertical Hyperbola:

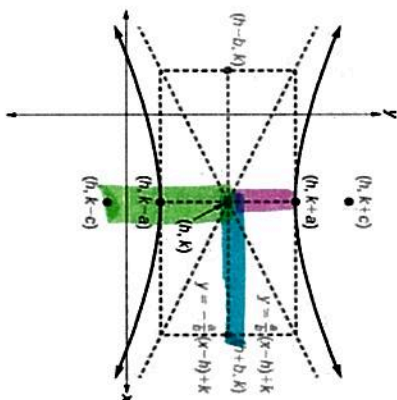
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

1st denom

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Equation to find foci: $c^2 = a^2 + b^2$

slopes of asymptotes = $\pm \frac{a}{b}$



1. Graph the hyperbola

$$\frac{(x+2)^2}{16} - \frac{(y-3)^2}{25} = 1$$

$a^2 = 16$

$b^2 = 25$

$b = 5$

Center: $(-2, 3)$

Vertices: $(-4, 3)$ and $(2, 3)$

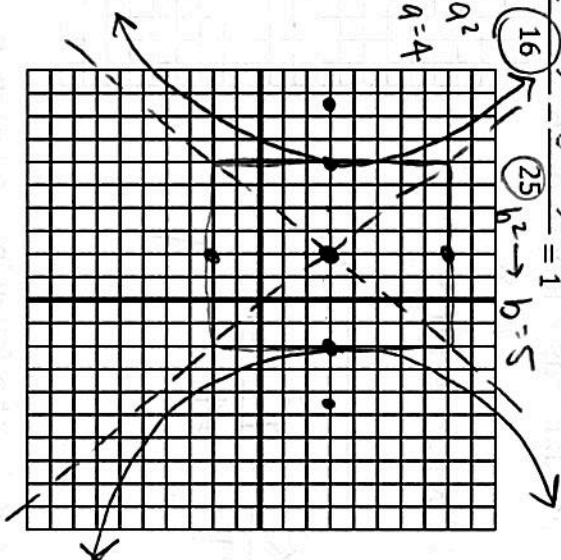
Asymptotes: Slopes $\pm 5/4$

Foci:

$(-8, 4)$ and $(4, 4)$

Length of Trans. Axis: 8

Length of Conj. Axis: 10



$$c^2 = 16 + 25$$

$$c^2 = 41 \rightarrow c = \sqrt{41} \approx 6.4$$

2. Graph the hyperbola

$$\frac{(y+1)^2}{9} - \frac{(x-2)^2}{16} = 1$$

$a^2 = 9$

$b^2 = 16$

Center: $(2, -1)$

Vertices: $(2, 2)$ and $(2, -4)$

Asymptotes: slopes $\pm 3/4$

Foci:

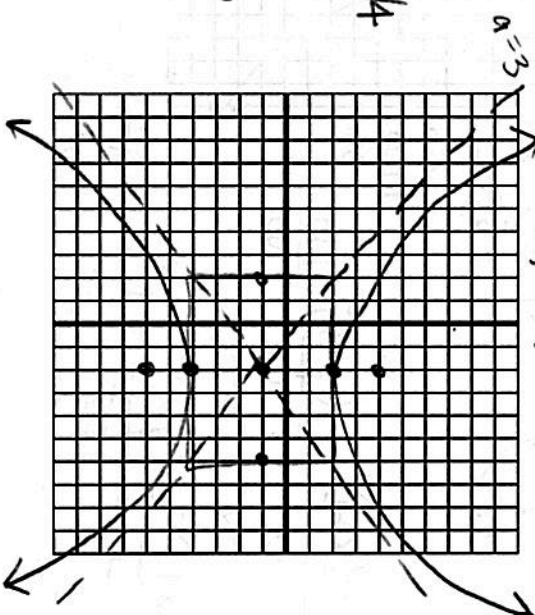
$(2, 4)$ and $(2, -6)$

Length of Trans. Axis: 6

Length of Conj. Axis: 8

$$c^2 = 9 + 16$$

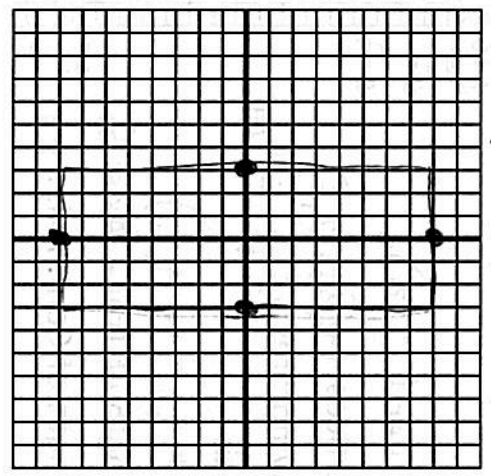
$$c^2 = 25 \rightarrow c = 5$$



3. Write an equation in standard form for the hyperbola whose vertices are at $(-3, 0)$ and $(3, 0)$, and whose conjugate axis has a length of 16 units. $a=3$

Center $(0,0)$
 $b=8$

$$\frac{x^2}{9} - \frac{y^2}{64} = 1$$



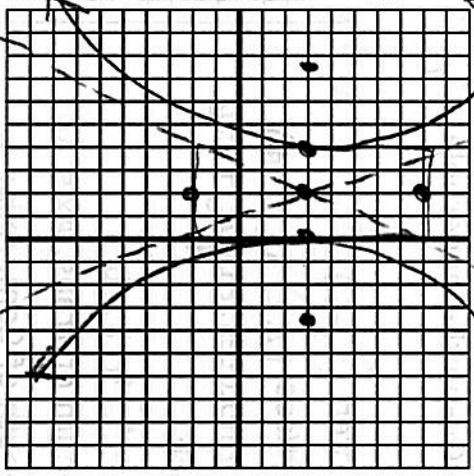
4. Write the following equation in standard form, then graph it.
 $25x^2 - 4y^2 + 100x + 24y - 36 = 0$

$$25x^2 + 100x - 4y^2 + 24y = 36$$

$$5\left(x^2 + \frac{4x+4}{1}\right) - 4\left(y^2 - \frac{6y+9}{2}\right) = 36$$

$$\frac{25(x+2)^2}{100} - \frac{4(y-3)^2}{100} = \frac{36}{100}$$

$$\frac{(x+2)^2}{4} - \frac{(y-3)^2}{25} = 1$$



Center $(-2, 3)$

$a^2 = 4 \rightarrow a = 2$
 $b^2 = 25 \rightarrow b = 5$
 $c^2 = 4 + 25 \rightarrow \sqrt{c^2} = \sqrt{29}$
 $c = \sqrt{29} \approx 5.4$

Foci: $(-1, 4, 3)$ & $(3, 4, 3)$

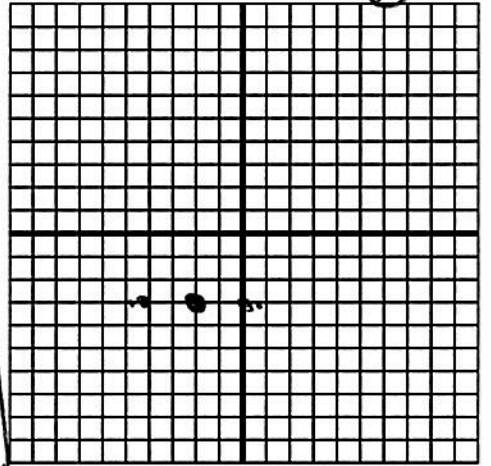
5. Find the coordinates of the vertices and foci for the hyperbola $3y^2 - 4x^2 + 12y + 24x = 36$

$$3y^2 + 12y - 4x^2 + 24x = 36$$

$$3\left(y^2 + \frac{4y+4}{1}\right) - 4\left(x^2 - \frac{6x+9}{2}\right) = 36$$

$$\frac{3(y+2)^2}{12} - \frac{4(x-3)^2}{12} = \frac{36}{12}$$

$$\frac{(y+2)^2}{4} - \frac{(x-3)^2}{3} = 1$$

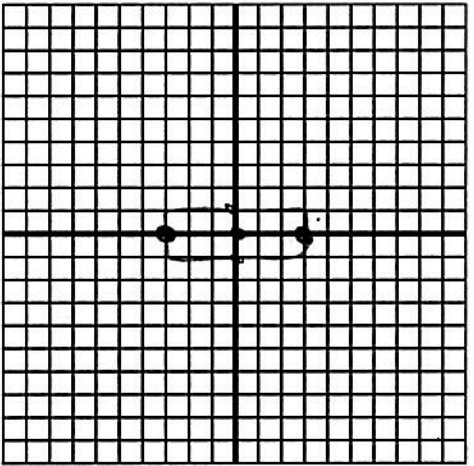


Center $(3, 2)$ $a=2$
 $b = \sqrt{3} \approx 1.7$
 $c^2 = 4 + 3 \rightarrow \sqrt{c^2} = \sqrt{7}$
 $c = \sqrt{7} \approx 2.6$

Vertices $(3, 0)$ & $(3, 4)$
 Foci $(3, 0.6)$ & $(3, 4.6)$

6. Find the standard form of the equation of the hyperbola with vertices at $(0, 3)$ and $(0, -3)$ and asymptotes $y = \pm 3x$

$$\frac{y^2}{9} - \frac{x^2}{1} = 1$$



Center $(0, 0)$
 $a=3$
 Asymp $\pm \frac{3}{1}x$