

## NOTES: Multiple Angle Formulas

### Solving a multiple-angle equation

1.  $2\cos x + \sin 2x = 0$

On the interval  $[0, 2\pi)$ .

$$2\cos x + 2\sin x \cos x = 0$$

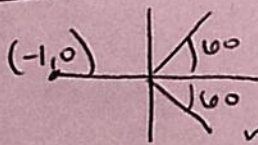
$$2\cos x(1 + \sin x) = 0$$

$1 + \sin x = 0 \rightarrow \sin x = -1$   
 $x = \frac{3\pi}{2}$

$2\cos x = 0 \rightarrow \cos x = 0$   
 $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$$2\cos^2 x - 1 + \cos x = 0$$

$$2\cos^2 x + \cos x - 1 = 0$$



$$(2\cos x - 1)(\cos x + 1) = 0$$

$\cos x = \frac{1}{2} \rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$   
 $\cos x = -1 \rightarrow x = \pi$

Simplify using the double angle formulas.

4.  $6\cos^2 x - 3$   
 $3(2\cos^2 x - 1)$   
 $3(\cos 2x)$   
 $3\cos(2x)$

### Application

6. Ignoring air resistance, the range of a projectile fired at an angle  $\theta$  with the horizontal and with an initial velocity of  $v_0$  feet per second is given by

$$r = \frac{1}{32} v_0^2 \sin 2\theta$$

where  $r$  is the horizontal distance (in feet) that the projectile will travel. A place kicker for a football team can kick a football from ground level with an initial velocity of  $\frac{80}{v_0}$  feet per second. At what angle must the player kick the football so that the football travels  $\frac{200}{r}$  feet?

$$200 = \frac{1}{32} (80)^2 \sin(2\theta)$$

$$\frac{200}{200} = \frac{200}{200} \sin(2\theta) \rightarrow \sin(2\theta) = 1$$

$$\frac{2\theta}{2} = \frac{\pi}{2} \rightarrow \theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

## DOUBLE ANGLE FORMULAS

$$\sin 2u = 2\sin u \cos u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2\cos^2 u - 1$$

$$= 1 - 2\sin^2 u$$

3.  $8\sin x \cos x = 2\sqrt{2}$

$$\frac{1}{4}(2\sin x \cos x) = \frac{2\sqrt{2}}{4}$$

$$\sin(2x) = \frac{\sqrt{2}}{2}$$

$$\frac{2x}{2} = \frac{\pi}{4}, \frac{3\pi}{4} \quad \text{Add } \pi!$$

$$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$$

5.  $(\cos x + \sin x)(\cos x - \sin x)$

$$\cos^2 x - \sin^2 x$$

$$\cos(2x)$$

