**NOTES: Multiple Angle Formulas**

**DOUBLE ANGLE FORMULAS**

$$\sin 2u = 2\sin u \cos u$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$= 2\cos^2 u - 1$$
$$= 1 - 2\sin^2 u$$

**Solving a multiple-angle equation**

1. $2\cos x + \sin 2x = 0$ on the interval $[0, 2\pi)$.

$2\cos x + 2\sin x\cos x = 0$

$2\cos x(1+\sin x) = 0$ → $1+\sin x = 0$, $\sin x = -1$, $x = \frac{3\pi}{2}$

$\frac{2}{2}\cos x = \frac{0}{2}$, $\cos x = 0$

2. $\cos 2x + \cos x = 0$ → $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$2\cos^2 x - 1 + \cos x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$

$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

3. $8\sin x\cos x = 2\sqrt{2}$

$\frac{1}{4}(2\sin x\cos x) = \frac{2\sqrt{2}}{4}$

$\sin(2x) = \frac{\sqrt{2}}{2}$

$\frac{2x}{2} = \frac{\frac{\pi}{4}, \frac{3\pi}{4}}{2}$ * Add $\pi$!

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

New Period: $\frac{2\pi}{2} = \pi$

**Simplify using the double angle formulas.**

4. $6\cos^2 x - 3$

$3(2\cos^2 x - 1)$

$3(\cos 2x)$

$3\cos(2x)$

5. $(\cos x + \sin x)(\cos x - \sin x)$

$\cos^2 x - \sin^2 x$

$\cos(2x)$

**Application**

6. Ignoring air resistance, the range of a projectile fired at an angle $\theta$ with the horizontal and with an initial velocity of $v_0$ feet per second is given by

$$r = \frac{1}{32}v_0^2 \sin 2\theta$$

where $r$ is the horizontal distance (in feet) that the projectile will travel. A place kicker for a football team can kick a football from ground level with an initial velocity of 80 feet per second. At what angle must the player kick the football so that the football travels 200 feet? ($v_0$ = 80, $r$ = 200, $\theta$)

$200 = \frac{1}{32}(80)^2 \sin(2\theta)$

$\frac{200}{200} = \frac{200}{200}\sin(2\theta)$ → $\sin(2\theta) = 1$

$\frac{2\theta}{2} = \frac{\frac{\pi}{2}}{2}$ → $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ * Add $\pi$!

New Period: $\frac{2\pi}{2} = \pi$