Pre-AP Precalculus

Name Key

Rational Functions Notes – Day 2
Graphing

1. **x- Intercepts-** where the graph crosses the x- axis. To find x-intercepts, let the numerator = 0 after you have reduced. X-intecepts are always written (x,0)

2. **y-intercepts-** where the graph crosses the y-axis. To find y-intercepts, simply let x = 0 and simplify your fraction. Y-intercepts are always written (o, y)

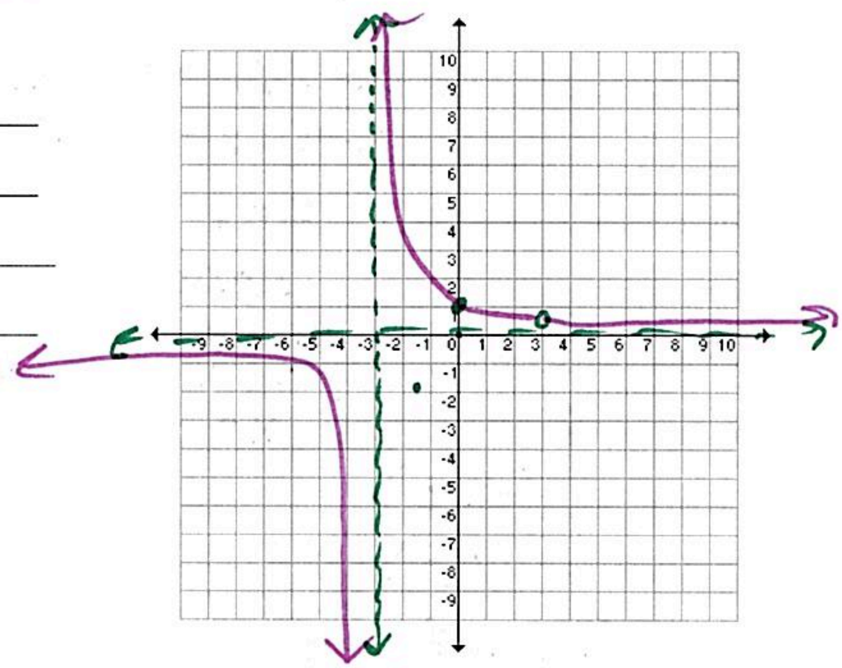
3. Although a rational function can never cross a vertical asymptote, SOMETIMES it can cross the horizontal asymptote or the slant asymptote. The function must approach the horizontal asymptotes as x $\to \infty$. To determine if a graph crosses the horizontal or slant asymptote, substitute the horizontal asymptote in for y in the equation, and solve for x. If you get an untrue statement, such as 1=5, then the graph will not cross the horizontal asymptote. But if you get an x value when you solve, such as x=3, then your graph will cross the horizontal asymptote at that point.

cross horiz asym? → set function equal to HA, solve for x.

1) $f(x) = \frac{3x-9}{x^2-9} = \frac{3\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \frac{3}{x+3}$

$0 = \frac{3}{x+3}$

$0 \neq 3$ not true



Horizontal asymptote: $y = 0$

Slant asymptote: none

Removable discontinuity: $(3, 1/2)$

Vertical asymptote: $x = -3$

x-intercept: none

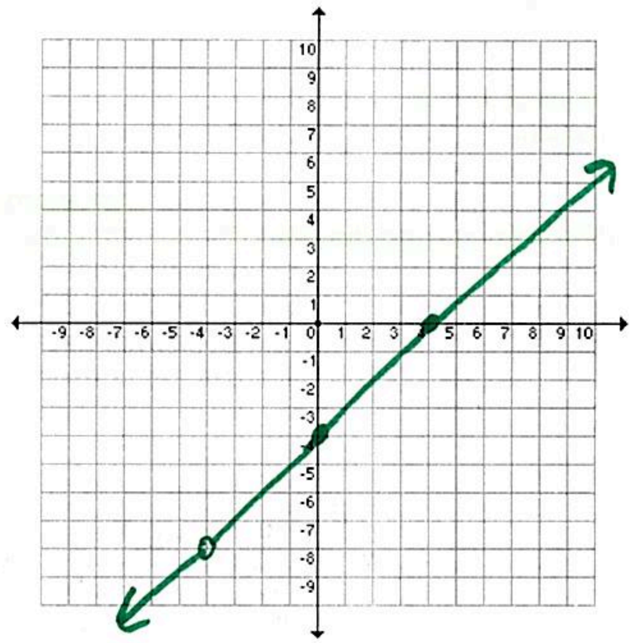
y-intercept: $(0, 1)$

domain: $\mathbb{R},\ x \neq 3, -3$

range: $\mathbb{R},\ y \neq 0, 1/2$

Does this graph cross the horizontal asymptote, and if so where? no

2) $f(x)=\frac{x^2-16}{x+4} = \frac{(x-4)\cancel{(x+4)}}{\cancel{x+4}} = x-4$ Linear!



Horizontal asymptote: none

Slant asymptote: none

Removable discontinuity: (−4, −8)

Vertical asymptote: none

x-intercept: (4, 0)

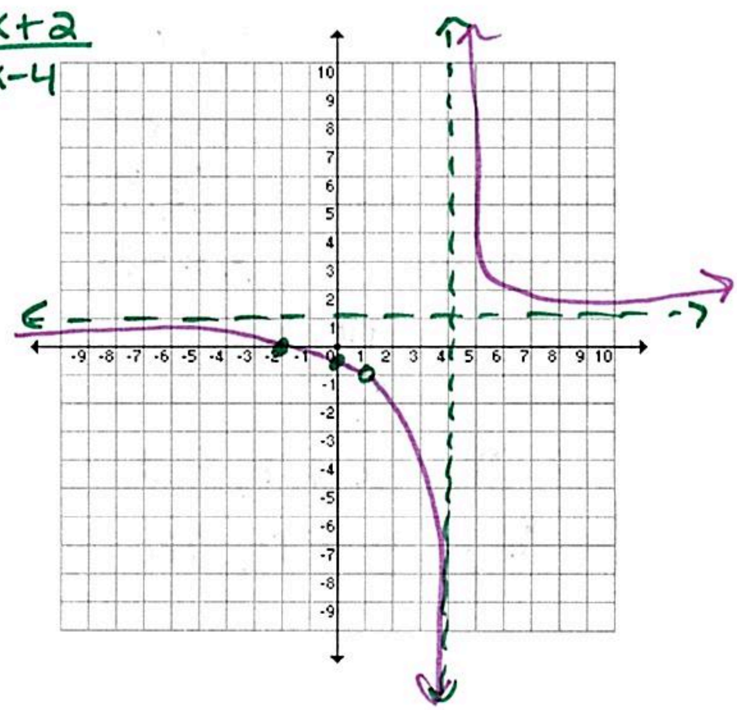
y-intercept: (0, −4)

domain: $\mathbb{R}$, $x \neq -4$

range: $\mathbb{R}$, $x \neq -8$

Does this graph cross the horizontal asymptote, and if so where? no

3) $f(x)=\frac{x^2+x-2}{x^2-5x+4} = \frac{(x+2)\cancel{(x-1)}}{(x-4)\cancel{(x-1)}} = \frac{x+2}{x-4}$



Horizontal asymptote: $y=1$

Slant asymptote: none

Removable discontinuity: (1, −1)

Vertical asymptote: $x=4$

x-intercept: (−2, 0)

y-intercept: (0, −1/2)

domain: $\mathbb{R}$, $x \neq 1, 4$

range: $\mathbb{R}$, $y \neq -1, 1$

Does this graph cross the horizontal asymptote, and if so where? no

$1=\frac{x+2}{x-4}$ $\quad x-4=\cancel{x}+2$

$-4 \neq 2$

4) $f(x)=\dfrac{2x^3-2x}{(x^3-x^2)-(9x+9)} = \dfrac{2x(x^2-1)}{x^2(x-1)-9(x-1)} = \dfrac{2x\cancel{(x-1)}(x+1)}{(x-3)(x+3)\cancel{(x-1)}} = \dfrac{2x(x+1)}{(x-3)(x+3)}$

Horizontal asymptote: $y=2$

Slant asymptote: none

Removable discontinuity: $(1,-1/2)$

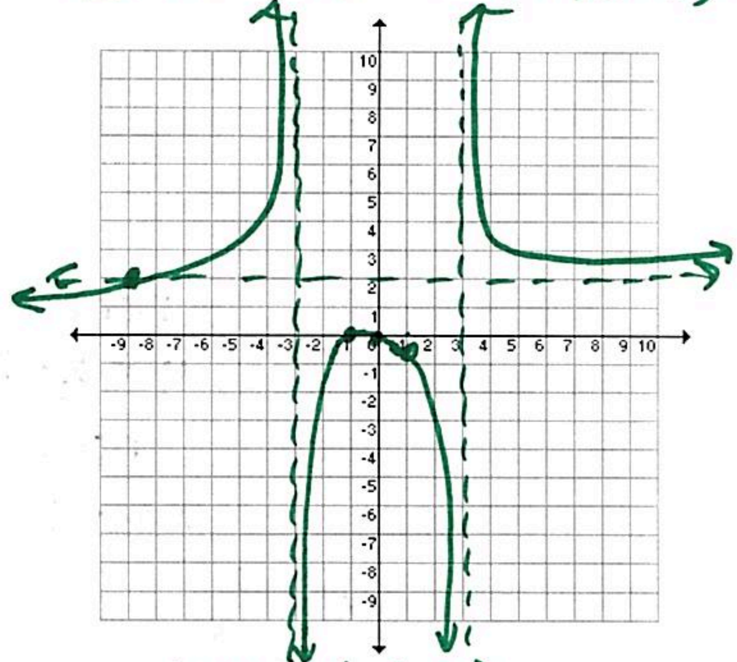
Vertical asymptote: $x=3,\ x=-3$

x-intercept: $(0,0)\ (-1,0)$

y-intercept: $(0,0)$

domain: $\mathbb{R},\ x\neq 3,-3,1$

range: can't tell



Does this graph cross the horizontal asymptote, and if so where? yes, $(-9,2)$

$2=\dfrac{2x(x+1)}{(x-3)(x+3)}$

$2(x^2-9)=2x^2+2x$

$\cancel{2x^2}-18=\cancel{2x^2}+2x$

$-18=2x \quad x=-9$ ✓

5) $f(x)=\dfrac{4x^2-8x-12}{(x^3-2x^2)-(9x+18)} = \dfrac{4(x^2-2x-3)}{x^2(x-2)-9(x-2)}$

$\dfrac{4\cancel{(x-3)}(x+1)}{\cancel{(x-3)}(x+3)(x-2)} = \dfrac{4(x+1)}{(x+3)(x-2)}$

Horizontal asymptote: $y=0$

Slant asymptote: none

Removable discontinuity: $(3,8/3)$

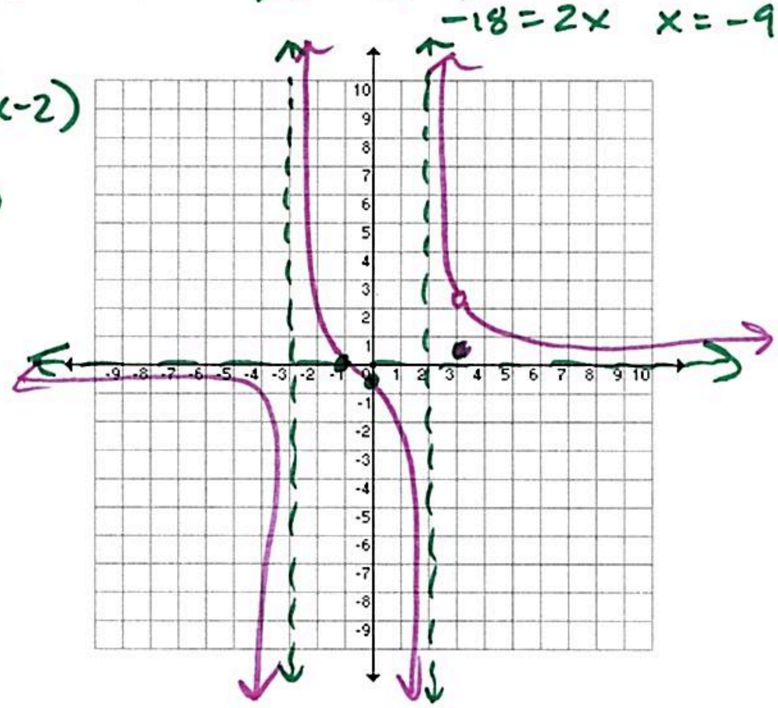
Vertical asymptote: $x=-3,\ x=2$

x-intercept: $(-1,0)$

y-intercept: $(0,-2/3)$

domain: $\mathbb{R},\ x\neq -3,2,3$

range: $\mathbb{R},\ y\neq 8/3$



Does this graph cross the horizontal asymptote, and if so where? yes, $(-1,0)$

$0=\dfrac{4(x+1)}{(x+3)(x-2)}$

$0=4x+4 \quad x=-1$ ✓

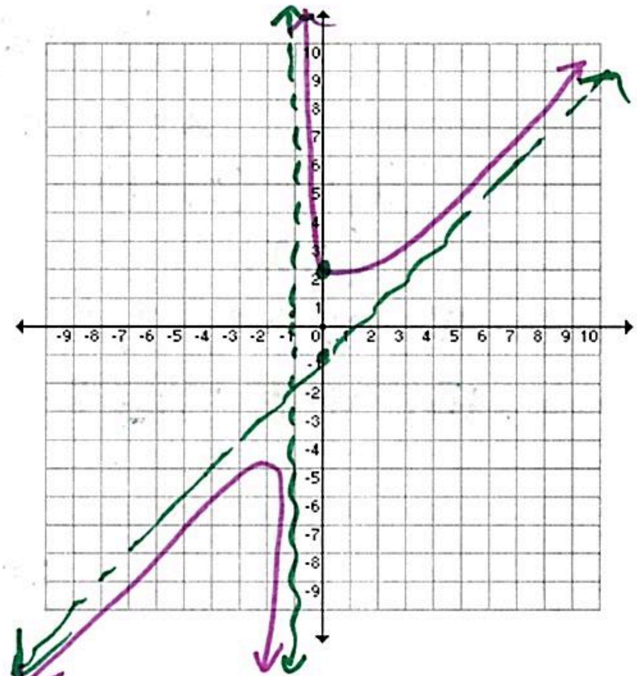
6) $f(x)=\dfrac{x^2+2}{x+1}$

$-1\,|\ 1\ \ 0\ \ 2$

$\quad\ -1\ \ 1$

$\quad\ 1\ -1\ \ X$

$y=x-1$



Horizontal asymptote: none

Slant asymptote: $y=x-1$

Removable discontinuity: none

Vertical asymptote: $x=-1$

x-intercept: none

y-intercept: $(0,2)$

domain: $\mathbb{R},\ x\neq -1$

range: can't tell

Does this graph cross the horizontal asymptote, and if so where? no

$$x-1=\frac{x^2+2}{x+1}$$

$$\cancel{x^2}-1=\cancel{x^2}+2$$

$$-1\neq 2$$