Lesson 3: Algebraic Techniques for Finding Limits

Limit Rules

Limit of a constant	$\lim\limits_{x\to\#} constant$ = constant	
Limit of a Sum/Difference	$\lim\limits_{x\to\#}[f(x) \pm g(x)]$	$= \lim\limits_{x\to\#} f(x) \pm \lim\limits_{x\to\#} g(x)$
Limit of a Product	$\lim\limits_{x\to\#}[f(x)\cdot g(x)]$	$= \lim\limits_{x\to\#}(f(x)) \cdot \lim\limits_{x\to\#}(g(x))$
Limit of a Quotient	$\lim\limits_{x\to\#} \frac{f(x)}{g(x)}$	$= \frac{\lim_{x\to\#} f(x)}{\lim_{x\to\#} g(x)}$ provided $\lim\limits_{x\to\#} g(x) \neq 0 \; and \; g(x) \neq 0$

Examples:

1. $\lim\limits_{x\to c} 7 =$ 7
2. $\lim\limits_{x\to c} x =$ c
3. $\lim\limits_{x\to 3}(x+4) =$ 7
4. $\lim\limits_{x\to 2}(x-5) =$ −3
5. $\lim\limits_{x\to 3}(-5x) =$ −15
6. $\lim\limits_{x\to 6}\frac{x}{2} =$ 3
7. $\lim\limits_{x\to 3}(5x^2) =$ 45
8. $\lim\limits_{x\to 2}(3x^4 - 5x^3 + 2x^2 - 6) =$ 10

Techniques for Evaluating Limits

Remember, if the function is continuous at the value indicated, use direct substitution.

If the function is NOT continuous at the value indicated:

Try 1) Factoring and reducing, 2) Long or synthetic division, 3) multiplying by the conjugate. 4) simplifying a complex expression, 5) using trig identities, or if all else fails, 6) sketch a graph of the function.

I. Dividing Out:

Factor and cancel where applicable. If you cannot factor it, then use synthetic or long division. Then use direct substitution on the simplified form (or quotient).

Ex 1: $\lim\limits_{x\to 2}\frac{x^3-2x^2-x+2}{x^4-2x^3+x-2}$

$$\frac{x^2\cancel{(x-2)}-1\cancel{(x-2)}}{x^3\cancel{(x-2)}-1\cancel{(x-2)}} = \frac{x^2-1}{x^3-1} = \frac{3}{9} = \boxed{\frac{1}{3}}$$

Ex 2: $\lim\limits_{x\to 4}\frac{2x^3-3x^2-23x+12}{x-4}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & \downarrow & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & \emptyset \end{array}$$

$$\lim_{x\to 4} 2x^2+5x-3 = \boxed{49}$$

II. Rationalizing Technique: Multiply by the conjugate to rationalize either the numerator **or** the denominator.

Ex 3: $\lim\limits_{x\to4}\dfrac{\sqrt{x}-2}{x-4}\cdot\dfrac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \dfrac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)} = \dfrac{1}{\sqrt{x}+2}$

$$\lim_{x\to4}\frac{1}{\sqrt{x}+2} = \boxed{\frac{1}{4}}$$

III. Using Algebra: Sometimes you have to use algebra and manipulate things:

Ex 4: $\lim\limits_{x\to2}\dfrac{\frac{4}{4}\cdot\frac{1}{x+2}-\frac{1}{4}\cdot\frac{(x+2)}{(x+2)}}{x-2} = \dfrac{\frac{4}{4(x+2)}-\frac{x+2}{4(x+2)}}{x-2} = \dfrac{\frac{2-x}{4(x+2)}}{\cancel{x-2}}\cdot\dfrac{1}{(x-2)}$

$$\frac{2-x}{4(x+2)(x-2)} = \frac{-1\cancel{(x-2)}}{4(x+2)\cancel{(x-2)}} = \frac{-1}{4(x+2)} = \boxed{-\frac{1}{16}}$$

IV. Trig When you see a trig function, and direct substitution does not work, try using identities:

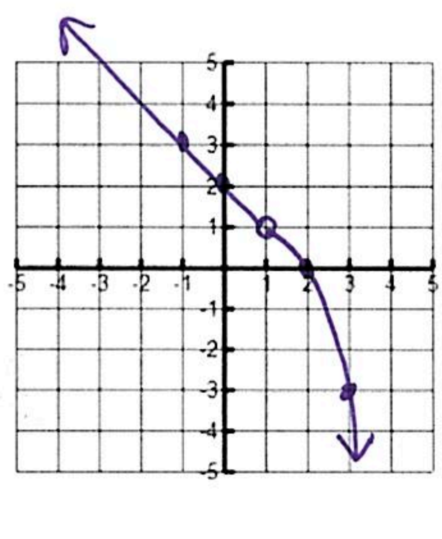
Ex 5: $\lim\limits_{x\to0}(sin^2x + cos^2x) =$ 1

$\lim\limits_{x\to0} 1 =$

Ex 6: $\lim\limits_{x\to0}(\csc x \sin x) =$ 1

$\dfrac{1}{\sin x}\cdot \sin x = 1$

Ex 7: Given $f(x)=\begin{cases} 2-x & x<1 \\ 2x-x^2 & x>1 \end{cases}$, find the following:



$\lim\limits_{x\to1^-} f(x) =$ 1

$\lim\limits_{x\to1^+} f(x) =$ 1

$\lim\limits_{x\to1} f(x) =$ 1