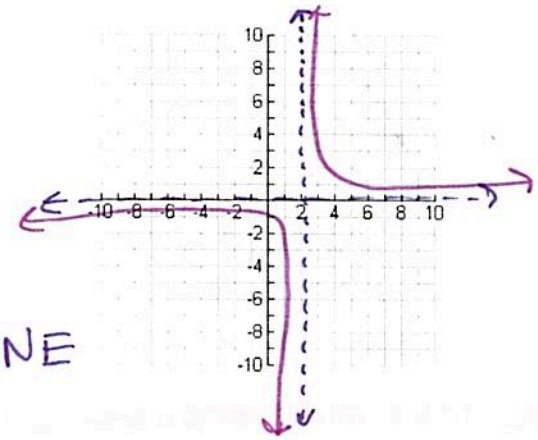
Lesson 4: Limits at Infinity

When can we do operations with infinity? (c is a constant greater than 0)

$\infty + \infty =$ ∞	$\infty + c =$ ∞
$\infty \times \infty =$ ∞	$\infty \times c =$ ∞
$\frac{\infty}{c} =$ ∞	$\frac{c}{\infty} =$ 0
$\infty - \infty =$?	$\frac{\infty}{\infty} =$?

A limit is a number by definition, so in general, limits that equal ∞ or -∞ do not exist, but we can use the infinite limits to describe end behavior.

1) Graph f(x) = $\frac{3}{x-2}$. Then find each limit.



a) $\lim_{x \to 2^-} \frac{3}{x-2}$ $-\infty$

b) $\lim_{x \to 2^+} \frac{3}{x-2}$ ∞

$\lim_{x \to 2} \frac{3}{x-2}$ = DNE

Vertical Asymptotes

The limit definition of a vertical asymptote is the x value where the $\lim_{x \to c} f(x) = \infty$ *or* $-\infty$ from the right or the left. If the limit is a number, there is no vertical asymptote.

Determine all vertical asymptotes of the graph of each function. Justify your answer using limits.

2) $f(x) = \frac{1}{2(x+1)}$ VA @ x = -1

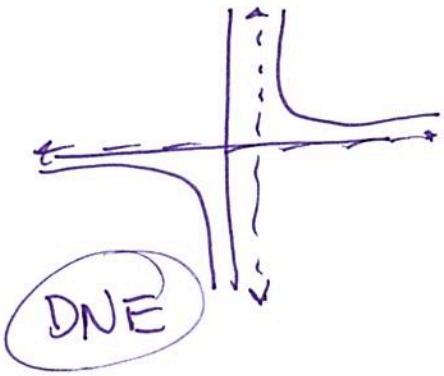
$\lim_{x \to -1^-} f(x) = -\infty$

$\lim_{x \to -1^+} f(x) = \infty$

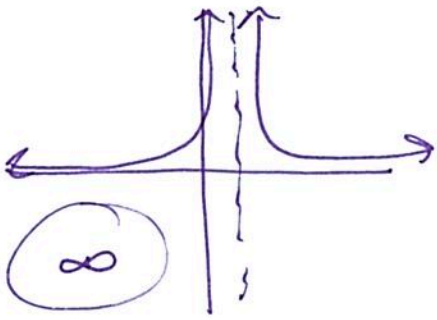
$\lim_{x \to -1} f(x)$ = DNE

Find each limit. (no calculator)

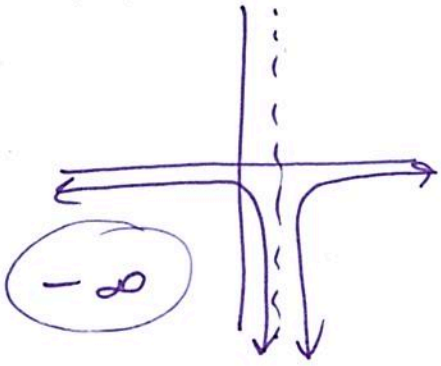
3) $\lim_{x\to 1}\frac{1}{x-1}$ $x=1$



4) $\lim_{x\to 1}\frac{1}{(x-1)^2}$

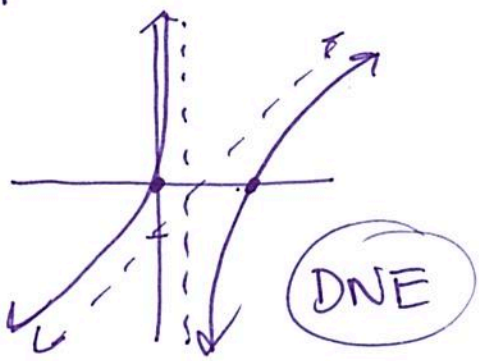


4) $\lim_{x\to 1}\frac{-1}{(x-1)^2}$



5) $\lim_{x\to 1}\frac{x^2-3x}{x-1} = \frac{x(x-3)}{x-1}$

BOTH



1 | 1 −3 0
↓ 1
1 −2
$y=x-2$

6) $\lim_{x\to 1}\frac{x^3-1}{x^2+x+1}$ $=\frac{(x-1)(x^2+x+1)}{x^2+x+1}$

$= x-1$

$\lim_{x\to 1} x-1 = \boxed{0}$

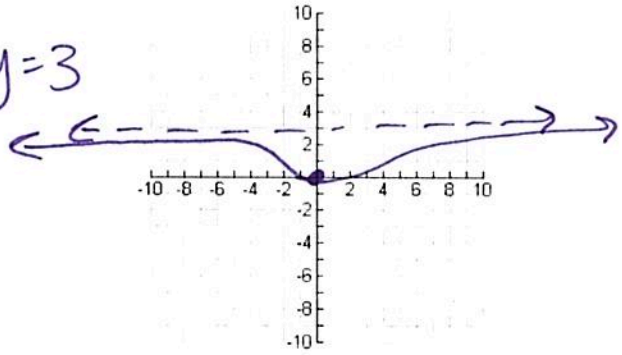
Linear!

Limits at Infinity

When finding a limit as $x\to\infty$ *or* $-\infty$, we only need to find the horizontal asymptote.

7) Graph $f(x)=\frac{3x^2}{x^2+1}$ then find each limit.

HA: $y=3$



a) $\lim_{x\to\infty}\frac{3x^2}{x^2+1}$ $\boxed{3}$

b) $\lim_{x\to-\infty}\frac{3x^2}{x^2+1}$ $\boxed{3}$

Guideline for finding limits at $\pm\infty$ of rational functions (horizontal asymptotes)

1) If the degree of the numerator is LESS THAN the degree of the denominator, then the limit of the rational function is ____0____. BOBO

2) If the degree of the numerator is EQUAL TO the degree of the denominator, then the limit of the rational function is the ____coefficient / coefficient____. COCC

3) If the degree of the numerator is GREATER THAN the degree of the denominator, then the limit of the rational function ____∞, $-\infty$, or DNE____. (which means the limit is either ∞ or $-\infty$, and you need continue the problem to determine that) BOTN

★ Plug in big numbers!

Find each limit. (no calculator)

8) $\lim_{x\to\infty}\left(5-\frac{2}{x^2}\right)$ $5-0=5$

9) $\lim_{x\to\infty}\frac{2x-1}{x+1}$ 2

10) $\lim_{x\to\infty}\frac{2x+5}{3x^2+1}$ 0

11) $\lim_{x\to\infty}\sqrt{\frac{2x^2+5}{3x^2+1}}$ $\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\sqrt{6}}{3}$

12) $\lim_{x\to\infty}\sqrt{\frac{2x^3+5}{3x^2+1}}$ ∞

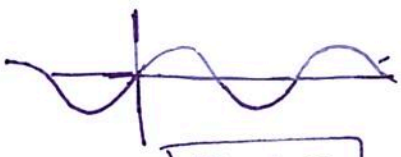
13) $\lim_{x\to\infty}\frac{2x^2-4x}{x+1}$ ∞

14) $\lim_{x\to\infty}\frac{x^{99}}{e^x}$ $=0$

★ Exponential is always bigger than a polynomial!

BOBO

15) $\lim_{x\to\infty}\sin x$



DNE

16) $\lim_{x\to-\infty}x^3$ $-\infty$