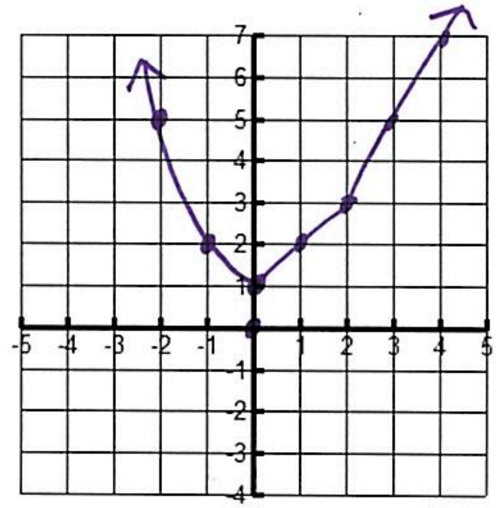
Lesson 2: Limits Introduction



Graph the piece-wise function: $y=\begin{cases} 2x-1 & \text{if } x>2 \\ x+1 & \text{if } 0\le x\le 2 \\ x^2+1 & \text{if } x<0 \end{cases}$

Domain: $\mathbb{R}$

Range: $[1,\infty)$ or $y\ge 1$

Continuous? yes If not, where? ______

Increasing? $(0,\infty)$ Decreasing? $(-\infty,0)$

What is a limit?

Notation: $\lim\limits_{x\to c} f(x)=L$

"the limit as x approaches c of f(x) equals L"

$\lim\limits_{x\to c^-} f(x)=$ the limit from the left

$\lim\limits_{x\to c^+} f(x)=$ the limit from the right

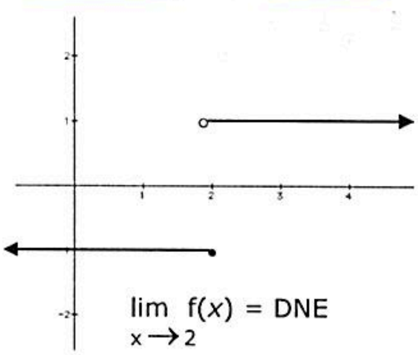
What does that mean?

As the x value gets closer and closer to some number c **from both sides**, the **y value** gets closer and closer to some number L.

> If the value of a function becomes close to a unique number ***L*** as ***x*** approaches a number ***c*** from both sides, the limit of the function as ***x*** approaches ***c*** is ***L.***
>
> $$\lim_{x\to c} f(x) = L$$

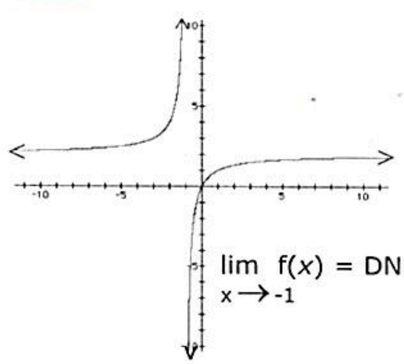
When $\lim\limits_{x\to c}$ **f(*x*)** DOES NOT EXIST – The limit of a function as x approaches c does not exist if :

*The function approaches different values from the left and right sides of **c**.



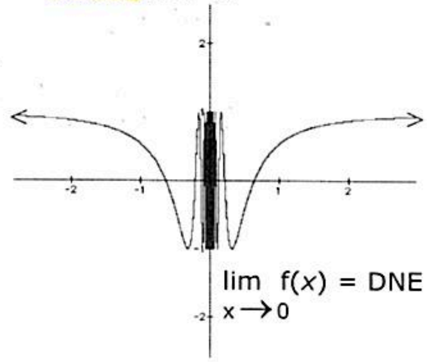
$\lim\limits_{x\to 2} f(x)$ = DNE

*The function has a a vertical asymptote at **c**.



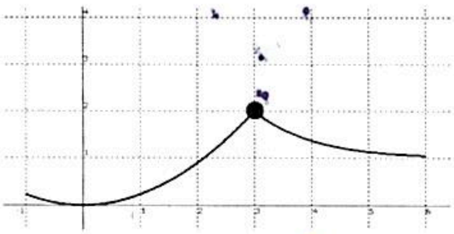
$\lim\limits_{x\to -1} f(x)$ = DNE

*The function oscillates between 2 different values near **c**.



$\lim\limits_{x\to 0} f(x)$ = DNE

Finding a limit graphically:

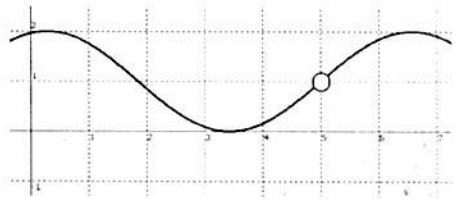


$\lim_{x\to 3^-} f(x) =$ 2

$\lim_{x\to 3^+} f(x) =$ 2

f(3) = 2

$\lim_{x\to 3} f(x) =$ 2

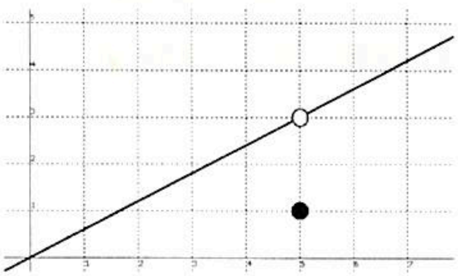


$\lim_{x\to 5^-} f(x) =$ 1

$\lim_{x\to 5^+} f(x) =$ 1

f(5) = DNE

$\lim_{x\to 5} f(x) =$ 1

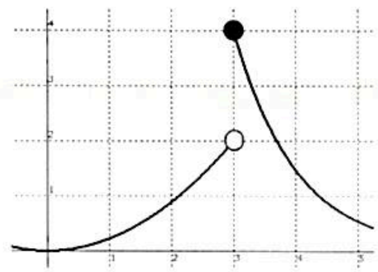


$\lim_{x\to 5^-} f(x) =$ 3

$\lim_{x\to 5^+} f(x) =$ 3

f(5) = 1

$\lim_{x\to 5} f(x) =$ 3



$\lim_{x\to 3^-} f(x) =$ 2

$\lim_{x\to 3^+} f(x) =$ 4

f(3) = 4

* $\lim_{x\to 3} f(x) =$ DNE

Different from left + right

If the function is continuous, you can find the limit of the function as x approaches c by finding f(c). This is called **direct substitution**.

Example 1: $\lim_{x\to 4}(3x^2) =$

$3(4)^2 =$

$3(16) = \boxed{48}$

Example 2: $\lim_{x\to 3}(x+3) =$

$3+3 = \boxed{6}$

Continuity:

Given a graph, a) state whether the function is continuous (C), or not continuous (NC)
b) if NC, state where it fails.

1. $f(x) = -\frac{x^3}{2}$ C

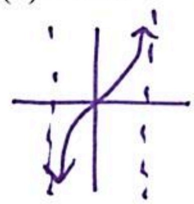
2. $f(x) = \frac{x^2-1}{x}$ NC @ $x=0$

3. $f(x) = \frac{x^2-1}{x+1}$ NC @ $x=-1$

4. $f(x) = \frac{1}{x^2-4}$ NC @ $x=\pm 2$

5. $f(x) = \begin{cases} x, & x<1 \\ 2, & x=1 \\ 2x-1, & 1<x \end{cases}$ NC jump @ $x=1$

6. $f(x) = \tan x$ NC @ $x = \frac{\pi}{2} + \pi K$



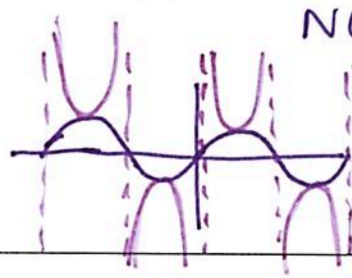
7. $f(x) = \frac{x^2}{x^2-36}$ NC @ $x=\pm 6$

8. $f(x) = x\sqrt{x+3}$ NC @ $x<-3$

9. $f(x) = \frac{x}{x^2+1}$ C

10. $f(x) = \frac{x+1}{\sqrt{x}}$ NC @ $x=0$

11. $f(x) = \csc x$ NC @ $x = \pi K$



12. $f(x) = \cot x$ NC @ $x = \pi + \pi K$

Radical simplification Review:

Simplify. Answers should be in simplified radical form with rational denominators.

a) $\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$

$= 3\sqrt{2}$

b) $\frac{5}{3\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$

$\frac{5\sqrt{7}}{21}$

c) $\frac{5}{(\sqrt{2}+3)} \cdot \frac{(\sqrt{2}-3)}{(\sqrt{2}-3)}$

$\frac{5\sqrt{2}-15}{2-9}$

$\boxed{\frac{5\sqrt{2}-15}{-7}}$

d) $\frac{\sqrt{3}}{\sqrt{2}-\sqrt{3}} \cdot \frac{(\sqrt{2}+\sqrt{3})}{(\sqrt{2}+\sqrt{3})}$

$\frac{\sqrt{6}+3}{2-3}$

$\boxed{\frac{\sqrt{6}+3}{-1}}$