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Nonlinear Inequalities

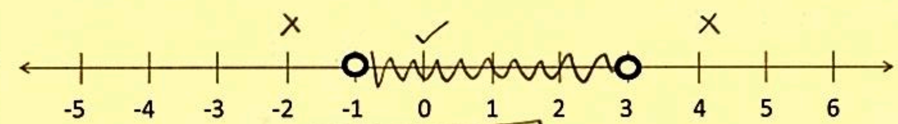
Polynomial Inequalities

To solve a polynomial inequality such as $x^2 - 2x - 3 < 0$, you can use the fact that a polynomial can only change signs at its zeroes (roots). Between two consecutive zeroes, a polynomial must be entirely positive or entirely negative. We will call these intervals ***test intervals***.

Find the zeroes for the polynomial (by factoring) in the inequality $x^2 - 2x - 3 < 0$ and mark them on the number line. These are called <u>critical numbers</u>.

Want < 0

$x^2 - 2x - 3 = 0 \rightarrow (x-3)(x+1) = 0 \rightarrow$ Roots: $x = 3$, $x = -1$



$\boxed{(-1, 3)}$

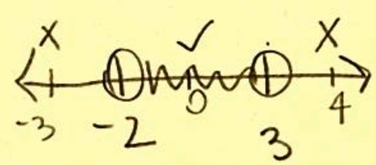
Now choose convenient values between your critical numbers to test the value of the expression $x^2 - 2x - 3$. Note whether your result is positive or negative for each test interval.

For which test interval(s) was the result <u>negative</u>? This is / these are the interval(s) where $x^2 - 2x - 3 < 0$.

1) <u>Solve</u>: $x^2 - x - 6 < 0$

$(x-3)(x+2) < 0$ want $(-)$

$x = 3 \quad x = -2$



$\boxed{(-2, 3)}$

2) <u>Solve</u>: $2x^3 - 3x^2 - 32x > -48$

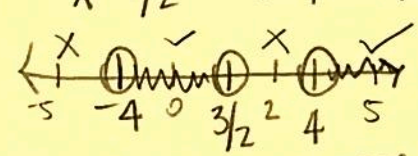
(Hint: factor by grouping)

$2x^3 - 3x^2 - 32x + 48 > 0$ want $(+)$

$x^2(2x-3) - 16(2x-3)$

$(2x-3)(x+4)(x-4) > 0$

$x = 3/2 \quad x = -4 \quad x = 4$



$\boxed{(-4, 3/2) \cup (4, \infty)}$

Unusual Solution Sets

A) The solution set to a polynomial inequality can be all real numbers, or (-∞, ∞). This occurs when the function lies completely above or below the horizontal axis.

B) Solution set can consist of a single real number. The number will be a critical number.

C) The solution set can be empty (no solution).

D) The solution set can consist of all real numbers with an exception, such as $x \neq 3$, which can be written $(-\infty, 3) \cup (3, \infty)$.

3) Solve: $x^2 + 2x + 4 > 0$ want above

Graph is always > 0

$\boxed{\mathbb{R}}$

4) Solve: $x^2 + 2x + 1 \le 0$ want below or on

$(x+1)(x+1) \le 0$

$x = -1$

-2 -1 0

$\boxed{-1}$

5) Solve: $x^2 + 3x + 5 < 0$ want below

Whole graph is above x-axis, not below

$\boxed{\text{No solution}}$

6) Solve: $x^2 - 4x + 4 > 0$ want above

$(x-2)(x-2)$

0 2 3

$\boxed{\mathbb{R},\ x \neq 2}$

or

$\boxed{(-\infty, 2) \cup (2, \infty)}$

Rational Inequalities

VA → always open circle!

Critical numbers are also used to find solutions sets for rational inequalities. Rational expressions change sign at their zeroes AND at their vertical asymptotes.

7) Find the solution set to $\frac{x+3}{x-2} > 0$.

VA: $x = 2$

x-int: $(-3, 0)$

-3 0 2

$\boxed{(-\infty, -3) \cup (2, \infty)}$

8) Find the solution set to $\frac{2x-7}{x-5} \le 3$.

$\frac{2x-7}{x-5} - \frac{3(x-5)}{1(x-5)} \le 0$ $\quad -3x+15$

$\frac{-x+8}{x-5} \le 0$

VA: $x = 5$

x-int: $(8, 0)$

5 8

$\boxed{(-\infty, 5) \cup [8, \infty)}$

9) Solve: $\frac{1}{x} - \frac{4x}{x} < 0$

$\frac{1-4x}{x} < 0$ $\quad \boxed{(-\infty, 0) \cup (1/4, \infty)}$

VA: $x = 0$

xint: $(1/4, 0)$

0 1/4

10) Solve: $\frac{x+12}{x+2} - \frac{3(x+2)}{x+2} \ge 0$

$\frac{x+12-3x-6}{x+2} \ge 0$

$\frac{-2x+6}{x+2} \ge 0$

VA: $x = -2$

x-int: $(3, 0)$

-2 3

$\boxed{(-2, 3]}$

11) Solve: $\frac{1}{x} \ge \frac{1}{x+3}$

$\frac{1(x+3)}{x(x+3)} - \frac{1(x)}{x+3(x)} \ge 0$

$\frac{x+3-x}{x(x+3)} \ge 0$ $\quad \boxed{(-\infty, -3) \cup (0, \infty)}$

VA: $x = 0, -3$

x-int: None

12) Solve: $\frac{5}{x-6} > \frac{3}{x+2}$

$\frac{5(x+2)}{x-6(x+2)} - \frac{3(x-6)}{x+2(x-6)} > 0$

$\frac{5x+10-3x+18}{(x-6)(x+2)} > 0$

$\frac{2x+28}{(x-6)(x+2)} > 0$ $\quad \boxed{(-14, -2) \cup (6, \infty)}$

VA: $x = 6, -2$