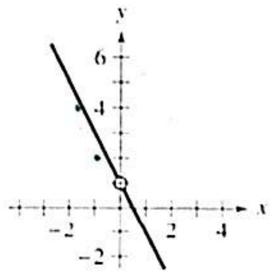
12.2 Limits Algebraically
12.4 Infinite Limits

Name Key Per_____

Use the graph to determine each limit (if it exists)

1. $g(x) = \dfrac{-2x^2 + x}{x}$

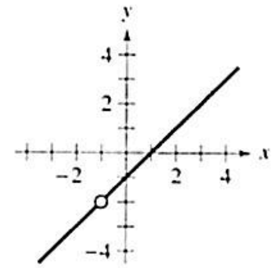


a) $\lim\limits_{x \to 0} g(x)$ $\boxed{1}$

b) $\lim\limits_{x \to -1} g(x)$ $\boxed{2}$

c) $\lim\limits_{x \to -2} g(x)$ $\boxed{4}$

2. $f(x) = \dfrac{x^2 - 1}{x + 1}$



a) $\lim\limits_{x \to 1} f(x)$ $\boxed{0}$

b) $\lim\limits_{x \to 2} f(x)$ $\boxed{1}$

c) $\lim\limits_{x \to -1} f(x)$ $\boxed{-2}$

Find the limit (if it exists). Use a graphing calculator to verify your results.

3. $\lim\limits_{x \to 6} \dfrac{x-6}{x^2 - 36}$ $\dfrac{\cancel{x-6}}{\cancel{(x-6)}(x+6)}$ $\dfrac{1}{x+6}$ $\boxed{\dfrac{1}{12}}$

4. $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x-2}$ $\dfrac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}}$ $\boxed{12}$

5. $\lim\limits_{x \to 1} \dfrac{x^4 - 1}{x-1}$ $\dfrac{(x^2-1)(x^2+1)}{(x-1)}$ $\dfrac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{x-1}}$ $\boxed{4}$

6. $\lim\limits_{y \to 0} \dfrac{\sqrt{5+y} - \sqrt{5}}{y} \cdot \dfrac{(\sqrt{5+y} + \sqrt{5})}{(\sqrt{5+y} + \sqrt{5})}$ $\dfrac{\cancel{5}+\cancel{y}-\cancel{5}}{\cancel{y}(\sqrt{5+y}+\sqrt{5})} = \dfrac{1}{\sqrt{5+y}+\sqrt{5}}$ $\dfrac{1}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$ $\boxed{\dfrac{\sqrt{5}}{10}}$

7. $\lim\limits_{x \to 2} \dfrac{4 - \sqrt{18-x}}{x-2} \cdot \dfrac{(4+\sqrt{18-x})}{(4+\sqrt{18-x})}$ $\dfrac{\cancel{16-(18-x)}\ \cancel{x-2}}{(4+\sqrt{18-x})\ \cancel{x-2}}$ $\boxed{\dfrac{1}{8}}$

8. $\lim\limits_{x \to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{x}$ $4 \quad \dfrac{(x+4)}{(x+4)} = \dfrac{\cancel{4}-x-\cancel{4}}{4(x+4)} \cdot \dfrac{1}{\cancel{x}}$ $\dfrac{-1}{4(x+4)} = \boxed{\dfrac{-1}{16}}$

9. $\lim\limits_{x \to \infty} \dfrac{3}{x^2}$ $= \boxed{0}$

10. $\lim\limits_{x \to \infty} \dfrac{1-6x}{1+5x}$ $\boxed{\dfrac{-6}{5}}$

11. $\lim\limits_{x \to -\infty} \dfrac{e^x}{1-x^2}$ $= \dfrac{1}{e^{-\infty}(1-x^2)}$

BOBO

$\boxed{0}$

12. $\lim_{y\to\infty} \frac{4y^4}{y^2+3}$ $\boxed{\infty}$

13. $\lim_{x\to-\infty} \frac{-(x^2+3)}{(2-x)^2}$ $\boxed{-1}$

14. $\lim_{x\to\infty}\left[7+\frac{2\cancel{x^2}}{(\cancel{x}+3)^2}\right]$ 2 $\boxed{9}$

15. $\lim_{x\to\infty} \frac{3x}{1-x}$ $\boxed{-3}$

16. $\lim_{x\to\infty} \frac{2x+1}{x^2-1}$ $\boxed{0}$

17. $\lim_{x\to\infty}\left(2+\frac{1}{\cancel{x}}\right)$ 0 $\boxed{2}$

18. $\lim_{x\to\frac{\pi}{2}} \csc x$

$\csc\frac{\pi}{2} = \boxed{1}$

$\sin\frac{\pi}{2} = 1$

19. $\lim_{x\to\pi} \cos x$

$\cos\pi = \boxed{-1}$

20. $\lim_{x\to-2} \frac{2x^3+7x^2+10x+8}{x+2}$

$$\begin{array}{r|rrrr} -2 & 2 & 7 & 10 & 8 \\ & \downarrow & -4 & -6 & -8 \\ \hline & 2 & 3 & 4 & 0 \end{array}$$

$2x^2+3x+4$

$2(-2)^2+3(-2)+4$

$8-6+4$

$\boxed{6}$

21. $\lim_{x\to\infty}\left(\sqrt{\frac{x^2-3x}{2x^2+5}}\right)$

$\frac{\sqrt{1}}{\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

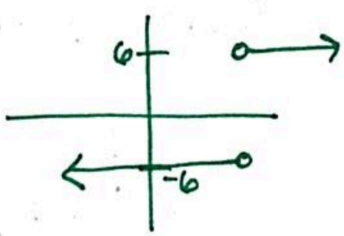
or $\boxed{\frac{\sqrt{2}}{2}}$

22. $\lim_{x\to\infty}\left(\frac{-3\cancel{x^2}}{\cancel{x^2}+2}+\frac{5\cancel{x^3}}{\cancel{x^4}}\right) = \boxed{-3}$

$-3+0$

23-25, Find the limit from the left, from the right, and then the overall limit.

23. $\lim_{x\to6} \frac{|x-6|}{x-6}$

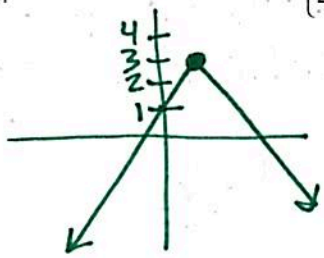


$\lim_{x\to6^-} = -1$

$\lim_{x\to6^+} = 1$

$\lim_{x\to6} = \text{DNE}$

24. $\lim_{x\to1} f(x)$, where $f(x)=\begin{cases}2x+1, & x<1\\ 4-x^2, & x\ge1\end{cases}$



$\lim_{x\to1^-} = 3$

$\lim_{x\to1^+} = 3$

$\lim_{x\to1} = 3$

25. $\lim_{x\to16} \frac{(4-\sqrt{x})}{x-16} \cdot \frac{(4+\sqrt{x})}{(4+\sqrt{x})}$

$\frac{16-x}{(x-16)(4+\sqrt{x})}$

$\frac{-1\cancel{(x-16)}}{\cancel{(x-16)}(4+\sqrt{x})}$

$= \frac{-1}{4+\sqrt{x}}$

$\lim_{x\to16^-} = -\frac{1}{8}$

$\lim_{x\to16^+} = -\frac{1}{8}$

$\lim_{x\to16} = -\frac{1}{8}$