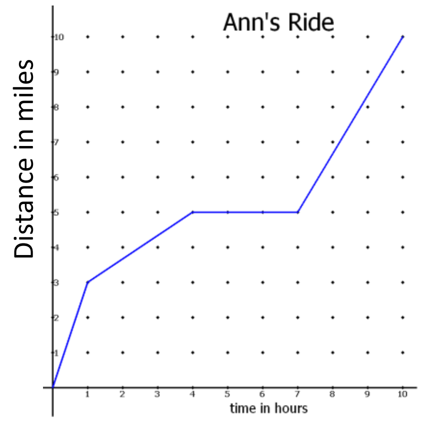
Lessson 1: Piecewise Functions

A piecewise function has different rules for different parts of its domain.

1. Ann went on a bicycle trip. The graph shows the relationship between time and distance traveled by Ann.

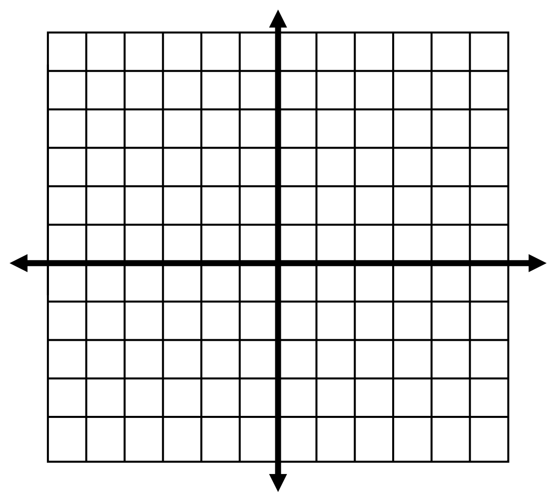


   a. Write a function *d(t)* for her distance in miles traveled in terms of the time *(t)* in hours.

$$d(t) = \begin{cases} \_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_ \end{cases}$$

   b. Find *d(3)*.

2. Graph the following piece-wise function and state its domain and range.

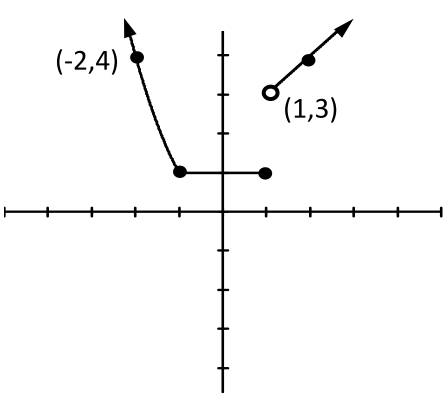


$$f(x) = \begin{cases} \frac{1}{2}x^2 & x \le 0 \\ \sqrt{x} & 0 < x < 4 \\ \frac{x}{4} & x \ge 4 \end{cases}.$$

Evaluate: *f(0)=* *f(4)=* *f(1)=*

D: __________________________ R: ______________________________

3. Write the piece-wise function for the graph shown.



$$f(x) = \begin{cases} \_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_\_\_, & \_\_\_\_\_\_\_\_ \end{cases}$$

Continuity of Functions

Continuous - The function is a single unbroken curve. You can trace the function from left to right without lifting your pencil.

Discontinuous - This is the result if there is one or more discontinuity.

- Removable Discontinuity (hole) - the function would be continuous in the absence of the hole. if you "plug" the hole, the function becomes continuous.
- Jump Discontinuity - At a certain domain value, the function's value "jumps" and then continues. The jump creates a break in the curve where you must lift your pencil to trace it.
- Infinite Discontinuity - A vertical asymptote interrupts the continuity of the function.

In order for a function to be continuous at a point,

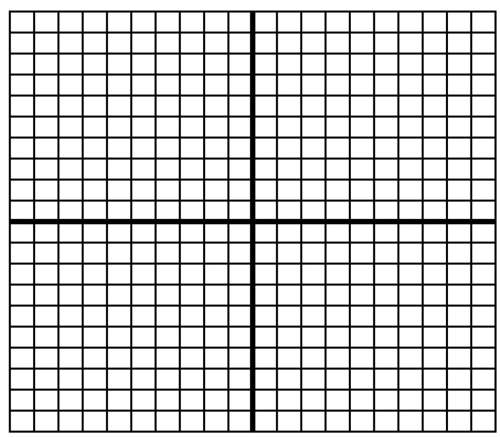
a) f(a) must exist

b) $\lim_{x \to a} f(x)$ must exist

c) f(a)= $\lim_{x \to a} f(x)$

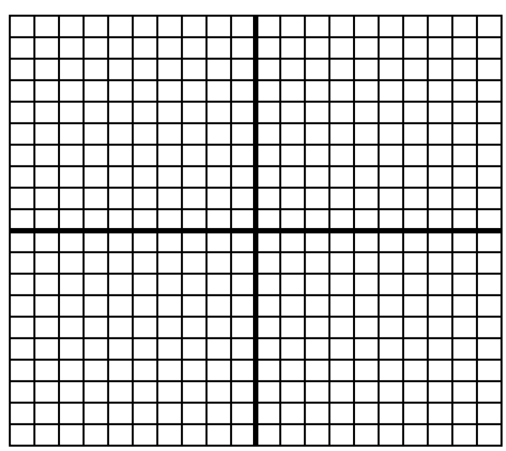
Sketch the graph of each function, then identify the types of discontinuities (if any).

4. $f(x) = \begin{cases} 1-(x-1)^2, & x \le 2 \\ \sqrt{x-2}, & x > 2 \end{cases}$



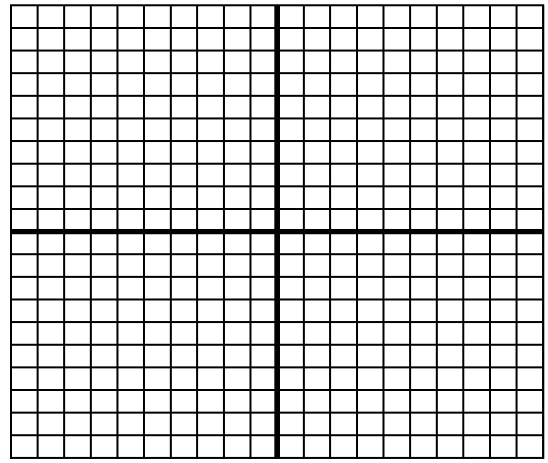
Discontinuities:______________________________

5. $f(x) = \begin{cases} x^2, & x \le 2 \\ 8-2x, & 2 < x < 4 \\ 4, & x \ge 4 \end{cases}$



Discontinuities:_______________________________

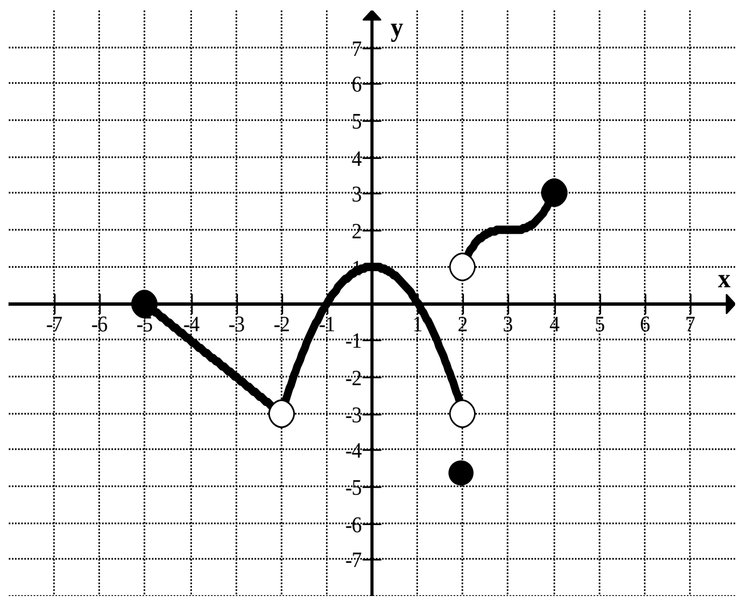
6. $f(x) = \begin{cases} x+2, & x \le 0 \\ \dfrac{1}{x}, & x > 0 \end{cases}$



Discontinuities:____________________________

7. Write a piecewise defined function for the following graph.

$f(x) = \begin{cases} \\ \\ \\ \end{cases}$



Find the following values:

f(-4) =

f(2) =

f(0) =

f(3) =

What is the absolute maximum? ____________ The absolute minimum? ____________

Find the interval(s) on which the function is increasing. ____________________________________

Find the interval(s) on which the function is decreasing. ____________________________________

Where is this function discontinuous within the domain [-5,4]? ____________________________________

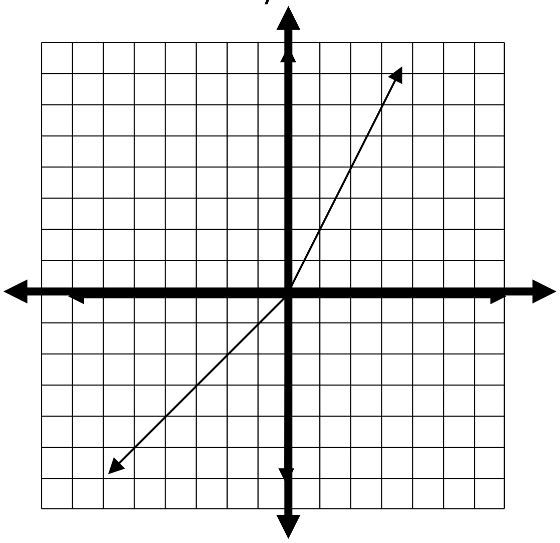
What is the rate of change on the interval (-5, -2)? ____________________________________

Where is the rate of change zero? ____________________________________

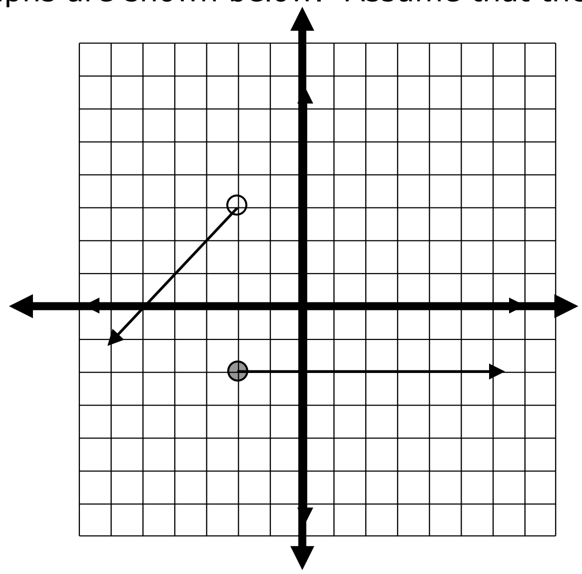
Piecewise Functions Homework

Write equations for the piecewise functions whose graphs are shown below. Assume that the units are 1 for every tick mark.

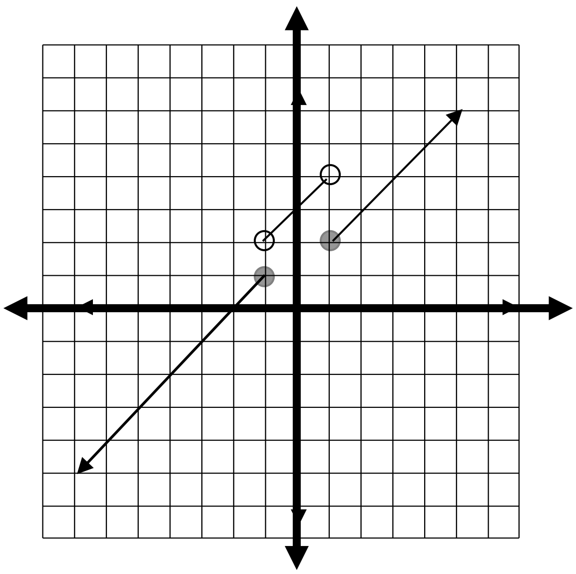
1.



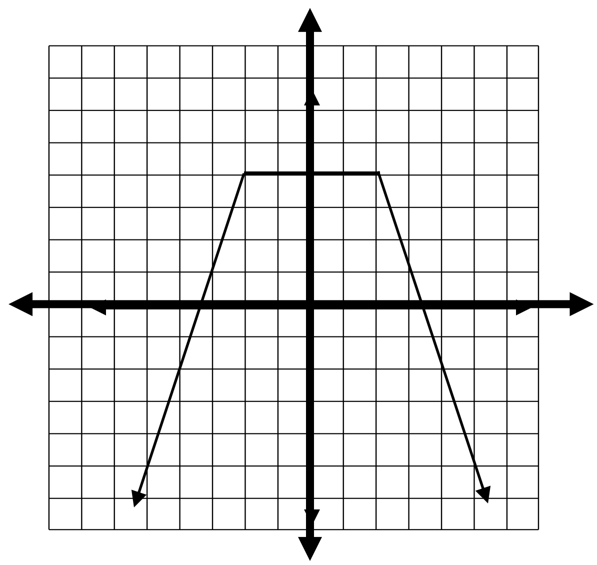
2.



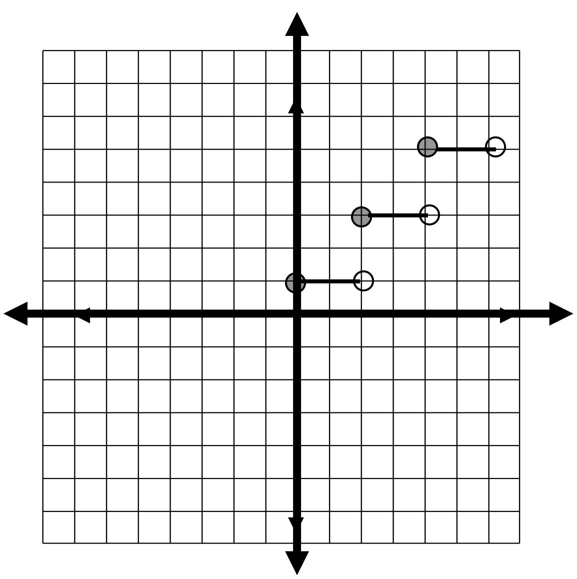
3.



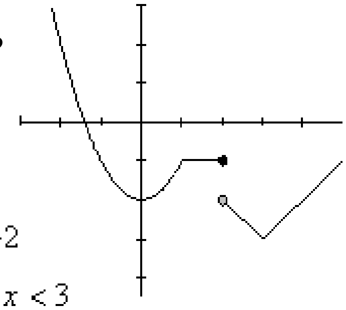
4.



5.



6. Which of the following piecewise functions is represented by this graph?



a. $f(x) = \begin{cases} x^2 - 2 & x \le 1 \\ -1 & 1 < x < 2 \\ |x-3| - 3 & x \ge 2 \end{cases}$

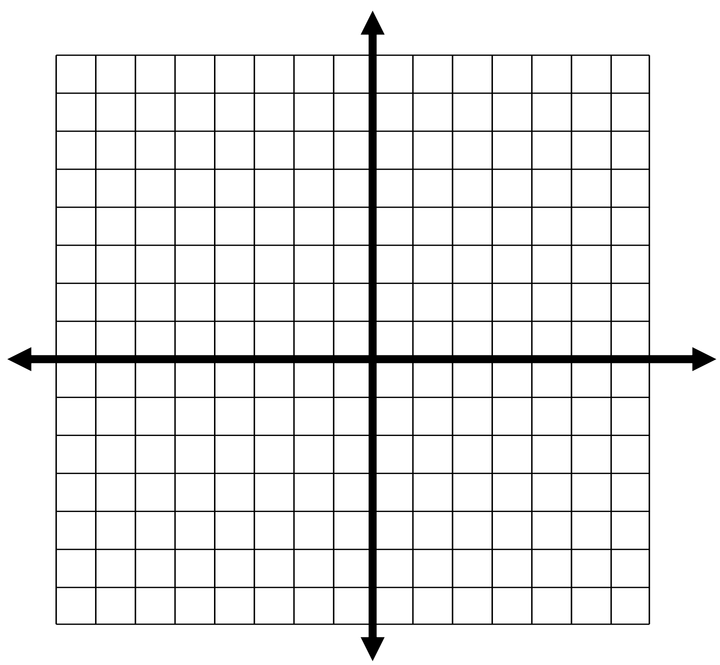
c. $f(x) = \begin{cases} x^2 - 2 & x \le -2 \\ -1 & -2 < x < 3 \\ |x-3| - 3 & x \ge 3 \end{cases}$

b. $f(x) = \begin{cases} x^2 - 2 & x \le -2 \\ -1 & -2 < x \le 3 \\ |x-3| - 3 & x > 3 \end{cases}$

d. $f(x) = \begin{cases} x^2 - 2 & x \le 1 \\ -1 & 1 < x \le 2 \\ |x-3| - 3 & x > 2 \end{cases}$

7. Graph the following piecewise function and then answer questions relating to it.

$$f(x) = \begin{cases} x+1, & x < -4 \\ 2, & -4 \le x < 0 \\ x^2 + 2, & x \ge 0 \end{cases}$$



Find the following values:

f(6) =

f(-4) =

f(0) =

f(3) =

What is the absolute maximum?

The absolute minimum?

Find the interval(s) on which the function is increasing.

Find the interval(s) on which the function is decreasing.

Find the interval(s) on which the function is constant.

Where is the function discontinuous?

What is the rate of change on the interval (-6, -4)?

Piecewise Functions Practice

Evaluate the function for the given value of x.

$$f(x) = \begin{cases} 3, & \text{if } x \le 0 \\ 2, & \text{if } x > 0 \end{cases} \qquad g(x) = \begin{cases} x + 5, & \text{if } x \le 3 \\ 2x - 1, & \text{if } x > 3 \end{cases} \qquad h(x) = \begin{cases} \frac{1}{2}x - 4, & \text{if } x \le -2 \\ 3 - 2x, & \text{if } x > -2 \end{cases}$$

1. $f(2)$ **2.** $f(-4)$ **3.** $f(0)$ **4.** $f\left(\frac{1}{2}\right)$

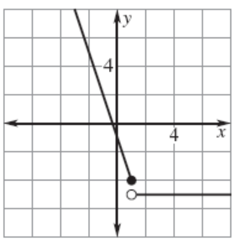
5. $g(7)$ **6.** $g(0)$ **7.** $g(-1)$ **8.** $g(3)$

9. $h(-4)$ **10.** $h(-2)$ **11.** $h(-1)$ **12.** $h(6)$

Match the piecewise function with its graph.

13. $f(x) = \begin{cases} x - 4, & \text{if } x \le 1 \\ 3x, & \text{if } x > 1 \end{cases}$ **14.** $f(x) = \begin{cases} x + 4, & \text{if } x \le 0 \\ 2x + 4, & \text{if } x > 0 \end{cases}$ **15.** $f(x) = \begin{cases} 3x - 2, & \text{if } x \le 1 \\ x + 2, & \text{if } x > 1 \end{cases}$

16. $f(x) = \begin{cases} 2x + 3, & \text{if } x \ge 0 \\ x + 4, & \text{if } x < 0 \end{cases}$ **17.** $f(x) = \begin{cases} 3x - 1, & \text{if } x \ge -1 \\ -5, & \text{if } x < -1 \end{cases}$ **18.** $f(x) = \begin{cases} -3x - 1, & \text{if } x \le 1 \\ -5, & \text{if } x > 1 \end{cases}$

A.


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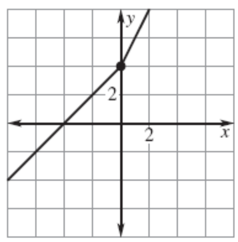

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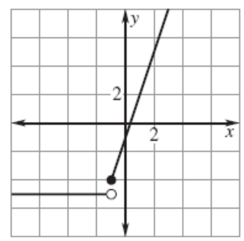
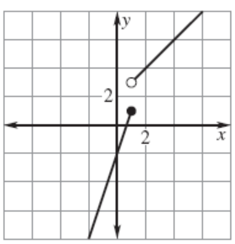


D.


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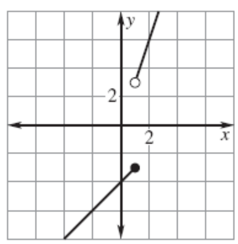

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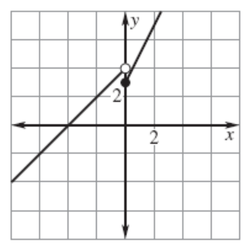


Graph the function.

19.

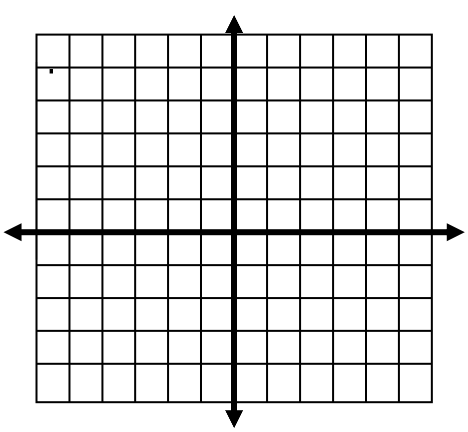
$$f(x) = \begin{cases} x + 3, & \text{if } x \le 0 \\ 2x, & \text{if } x > 0 \end{cases}$$

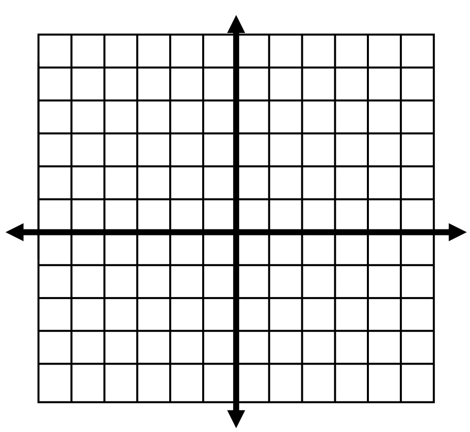
20.

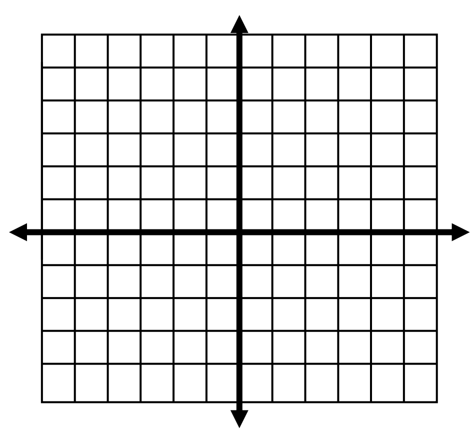
$$f(x) = \begin{cases} x + 1, & \text{if } x < 0 \\ -x + 1, & \text{if } 0 \le x \le 2 \\ x - 1, & \text{if } x > 2 \end{cases}$$

21.

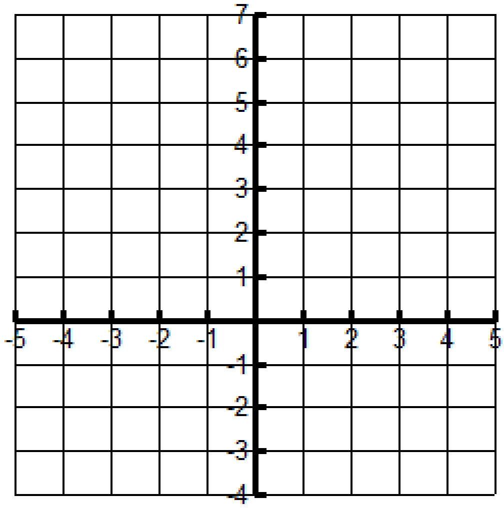
$$f(x) = \begin{cases} 2, & \text{if } x \le -3 \\ -1, & \text{if } -3 < x < 3 \\ 3, & \text{if } x \ge 3 \end{cases}$$







Lesson 2: Limits Introduction



Graph the piece-wise function: $y = \begin{cases} 2x - 1 & \text{if} \quad x > 2 \\ x + 1 & \text{if} \quad 0 \le x \le 2 \\ x^2 + 1 & \text{if} \quad x < 0 \end{cases}$

Domain: ________________

Range: __________________

Continuous? __________ If not, where? _____________________

Increasing? ____________ Decreasing? ____________________

What is a limit?

Notation: $\lim\limits_{x \to c} f(x) = L$

$\lim\limits_{x \to c^-} f(x) =$ the limit from the left

$\lim\limits_{x \to c^+} f(x) =$ the limit from the right

"the limit as x approaches c of f(x) equals L"

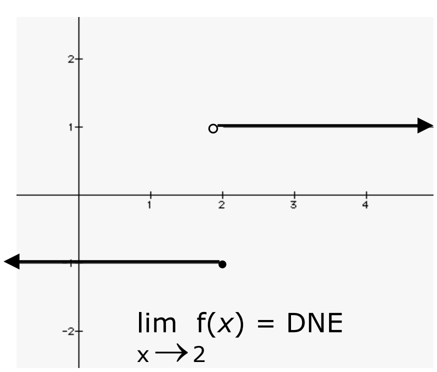
What does that mean?

As the x value gets closer and closer to some number c **from both sides**, the **y value** gets closer and closer to some number L.

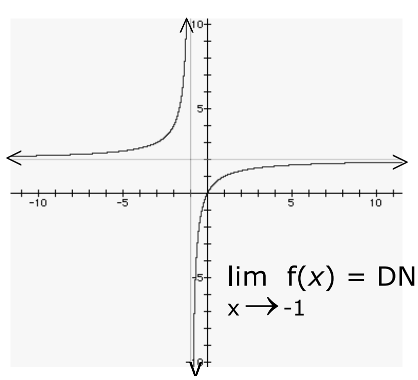
> If the value of a function becomes close to a unique number ***L*** as ***x*** approaches a number ***c*** from both sides, the limit of the function as ***x*** approaches ***c*** is ***L.***
>
> $$\lim_{x \to c} f(x) = L$$

When $\lim\limits_{x \to c}$ **f(x)** DOES NOT EXIST – The limit of a function as x approaches c does not exist if :

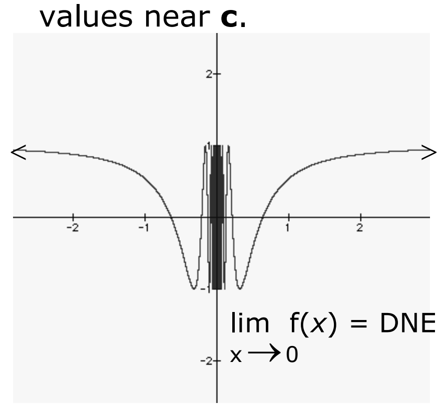
*The function approaches different values from the left and right sides of **c**.



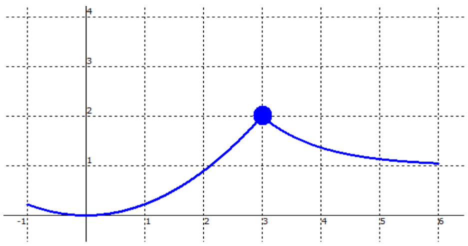
*The function has a vertical asymptote at **c**.



*The function oscillates between 2 different values near **c**.



Finding a limit graphically:

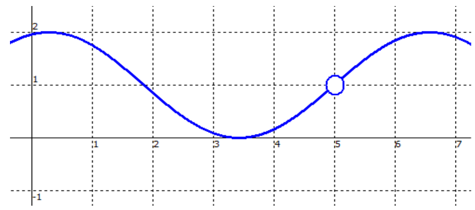


$\lim_{x\to 3^-} f(x) =$

$\lim_{x\to 3^+} f(x) =$

f(3) = ______

$\lim_{x\to 3} f(x) =$ _________

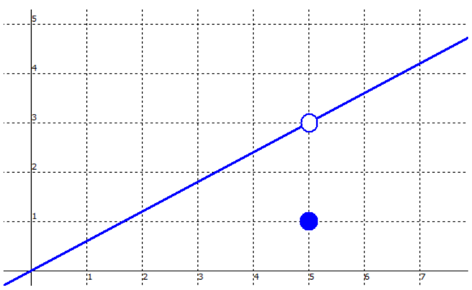


$\lim_{x\to 5^-} f(x) =$

$\lim_{x\to 5^+} f(x) =$

f(5) = _______

$\lim_{x\to 5} f(x) =$ _________

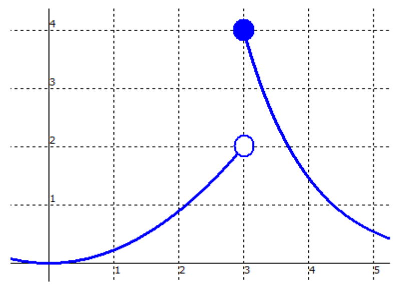


$\lim_{x\to 5^-} f(x) =$

$\lim_{x\to 5^+} f(x) =$

f(5) = ________

$\lim_{x\to 5} f(x) =$ _________



$\lim_{x\to 3^-} f(x) =$

$\lim_{x\to 3^+} f(x) =$

f(3) = ________

$\lim_{x\to 3} f(x) =$ ___________

If the function is continuous, you can find the limit of the function as x approaches c by finding f(c). This is called **direct substitution**.

Example 1: $\lim_{x\to 4}\left(3x^2\right) =$

Example 2: $\lim_{x\to 3}\left(x+3\right) =$

Continuity:

Given a graph, a) state whether the function is continuous (C), or not continuous (NC)
b) if NC, state where it fails.

1. $f(x) = -\frac{x^3}{2}$

2. $f(x) = \frac{x^2 - 1}{x}$

3. $f(x) = \frac{x^2 - 1}{x + 1}$

4. $f(x) = \frac{1}{x^2 - 4}$

5. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & 1 < x \end{cases}$

6. $f(x) = \tan x$

7. $f(x) = \frac{x^2}{x^2 - 36}$

8. $f(x) = x\sqrt{x + 3}$

9. $f(x) = \frac{x}{x^2 + 1}$

10. $f(x) = \frac{x + 1}{\sqrt{x}}$

11. f(x) = csc x

12. f(x) = cot x

Radical simplification Review:

Simplify. Answers should be in simplified radical form with rational denominators.

a) $\frac{6}{\sqrt{2}}$

b) $\frac{5}{3\sqrt{7}}$

c) $\frac{5}{\sqrt{2} + 3}$

d) $\frac{\sqrt{3}}{\sqrt{2} - \sqrt{3}}$

Limits Introduction
Homework

1. The graph of f(x) is shown in the figure below. Which of the following statements about f(x) is true?

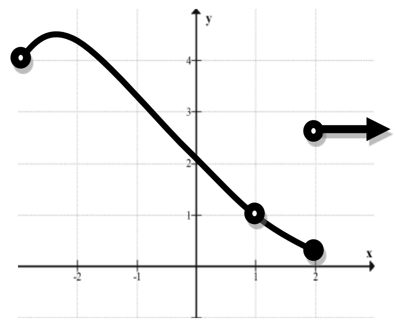
A) $\lim_{x \to 1} f(x) = DNE$

B) $\lim_{x \to 2} f(x) = 0.3$

C) $\lim_{x \to 2.01} f(x) < \lim_{x \to 0} f(x)$

D) $\lim_{x \to -1} f(x) \approx 3.3$

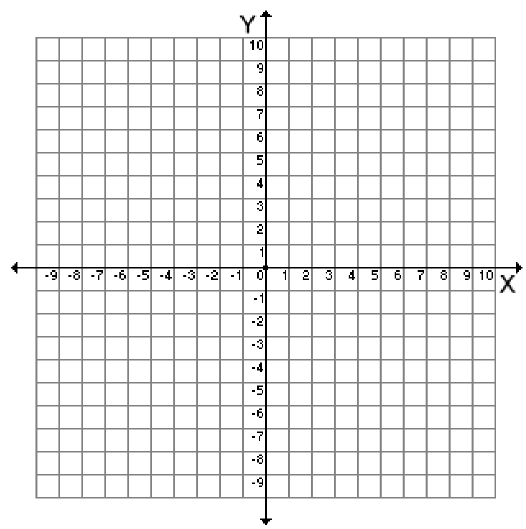
E) $\lim_{x \to -3} f(x) = 4$



2. Graph the function and find the limit (if it exists) as x approaches 2.

$$f(x) = \begin{cases} 2x+1, & x < 2 \\ x+3, & x \geq 2 \end{cases}$$

$\lim_{x \to 2} f(x) =$ _______



#3-8, Find the limit by direct substitution.

3. $\lim_{x \to 5} \left(10 - x^2\right) =$ ________________

4. $\lim_{x \to -3} \left(2x^2 + 4x + 1\right) =$ ________________

5. $\lim_{x \to \pi} \left(\sin \frac{x}{2}\right) =$ __________________

6. $\lim_{x \to 3} e^x =$ _________________

7. $\lim_{x \to \pi} \sec 2x =$ ___________________

8. $\lim_{x \to \pi} \tan\left(\frac{3x}{4}\right) =$ ________________

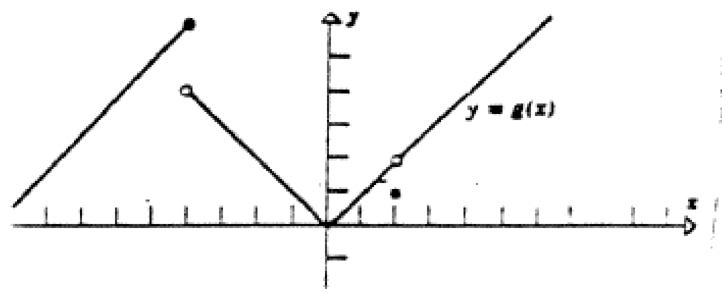
9. Find the limits using the piecewise function.

$\lim_{x \to -4^-} g(x) =$ ______

$\lim_{x \to -4^+} g(x) =$ ______

$\lim_{x \to 2} g(x) =$ ______

$g(2) =$ _______



10. Simplify. Answers should be in simplified radical form.

a) $\sqrt{\frac{18}{10}}$

b) $\frac{6}{\sqrt{5}+\sqrt{7}}$

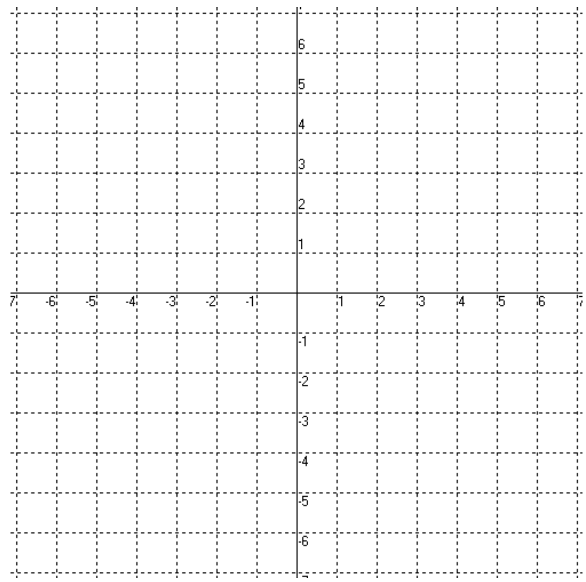
c) $\frac{5}{2\sqrt{2}}$

d) $\frac{6}{2\sqrt{3}}$

PreAP Precal
Practice with Piecewise and Intro to Limits

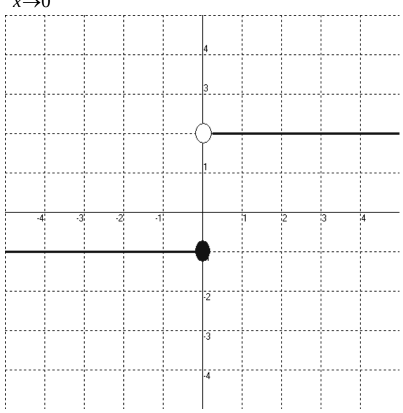
1. Graph the function. (Show a sketch of the graph.) Use the graph to find the indicated limit.

$$\lim_{x \to 0} f(x), \quad f(x) = \begin{cases} x - 1 & x < 0 \\ 3x\text{-}1 & x \geq 0 \end{cases}$$

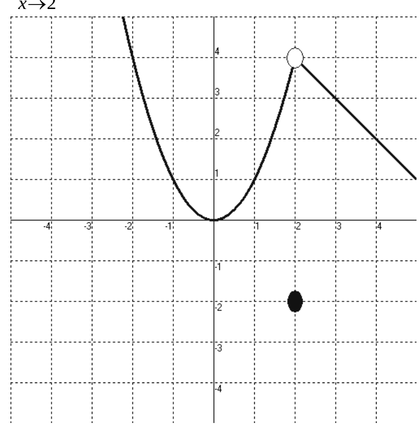


Give the limit of the following graph where f(x) is the graph shown.

2. $\lim_{x \to 0} f(x)$ =



3. $\lim_{x \to 2} f(x)$ =



Find the limit algebraically.

4. $\lim_{x \to 0} \dfrac{2 - x}{x^2 + 4} =$ _________

5. $\lim_{x \to -1} \dfrac{x^3 + x^2 + 3x + 3}{x^4 + x^3 + 2x + 2} =$ _________

6. $\lim_{x \to 2} \frac{|x-2|}{x-2} =$ ________

7. $\lim_{x \to -1} (4x^3 - 5x + 1) =$ ________

8. $\lim_{x \to -2^+} \frac{x-2}{x+2} =$ ________

9. $\lim_{x \to 4} \frac{2-\sqrt{x}}{4-x} =$ ________

10. The graph of the function f is shown in the figure. Which of the following statements about f is true?

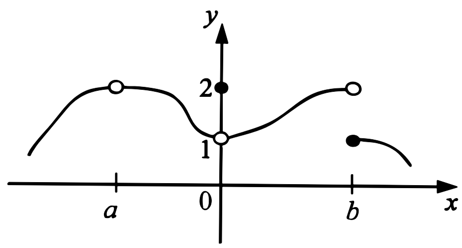
A) $f(a)$ exists

B) $\lim_{x \to a} f(x) = 2$

C) $\lim_{x \to b} f(x) = 1$

D) $\lim_{x \to b^-} f(x) = \lim_{x \to b^+} f(x)$

E) f is continuous at x=0



11. Evaluate the function below for the given value of x.

$$f(x) = \begin{cases} 9x - 4 & x > 3 \\ \frac{1}{2}x + 1 & x \le 3 \end{cases}$$

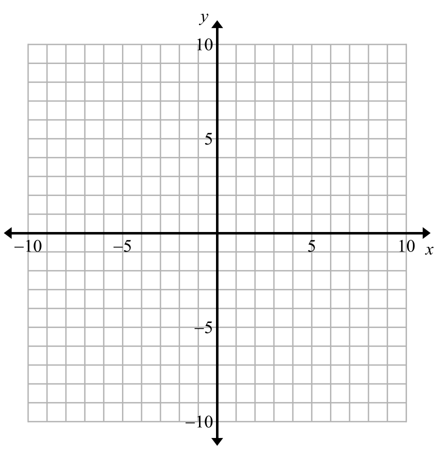
a) $f(-4)$.

b) $f(2)$.

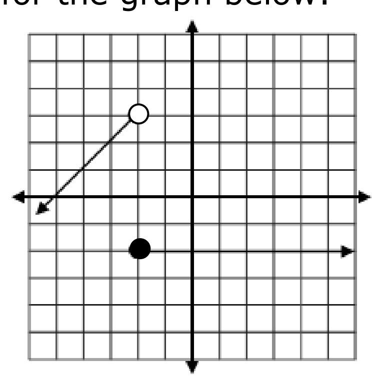
c) $f(3)$.

12. Graph the piecewise function:

$$f(x) = \begin{cases} -\frac{1}{2}x - 1 & x < 2 \\ 3x - 7 & x \ge 2 \end{cases}$$



13. Write a piecewise function for the graph below:



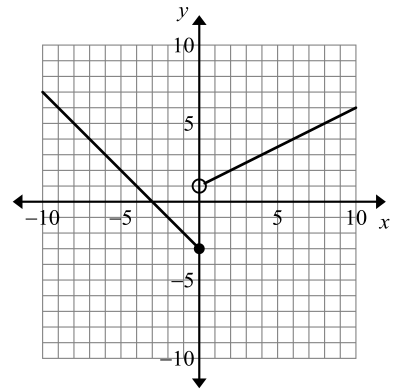
14. Which function is represented by the graph?

A. $f(x)=\begin{cases}-x-3, & \text{if } x\le 0\\ \frac{1}{2}x+1, & \text{if } x>0\end{cases}$

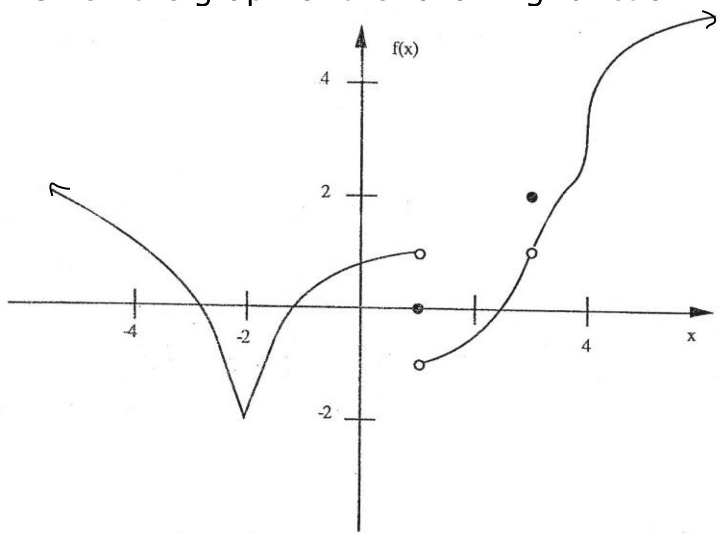
B. $f(x)=\begin{cases}x-3, & \text{if } x\le 0\\ -\frac{1}{2}x+1, & \text{if } x>0\end{cases}$

C. $f(x)=\begin{cases}-x+3, & \text{if } x\le 0\\ \frac{1}{2}x+1, & \text{if } x>0\end{cases}$

D. $f(x)=\begin{cases}x+3, & \text{if } x\le 0\\ -\frac{1}{2}x+1, & \text{if } x>0\end{cases}$



15. Given the graph of the following function.



Find the following:

A) $f(1)$

B) $\lim_{x\to 1^-} f(x)$

C) $\lim_{x\to 1^+} f(x)$

D) $\lim_{x\to 1} f(x)$

E) $f(3)$

F) $\lim_{x\to 3^-} f(x)$

G) $\lim_{x\to 3^+} f(x)$

H) $\lim_{x\to 3} f(x)$

I) $\lim_{x\to -2} f(x)$

J) $\lim_{x\to 0} f(x)$

K) $\lim_{x\to -3} f(x)$

Lesson 3: Algebraic Techniques for Finding Limits

Limit Rules

Limit of a constant	$\lim_{x\to\#} constant$ = constant		
Limit of a Sum/Difference	$\lim_{x\to\#}[f(x) \pm g(x)]$	=	$\lim_{x\to\#} f(x) \pm \lim_{x\to\#} g(x)$
Limit of a Product	$\lim_{x\to\#}[f(x) \cdot g(x)]$	=	$\lim_{x\to\#}(f(x)) \cdot \lim_{x\to\#}(g(x))$
Limit of a Quotient	$\lim_{x\to\#} \frac{f(x)}{g(x)}$	=	$\frac{\lim_{x\to\#} f(x)}{\lim_{x\to\#} g(x)}$ provided $\lim_{x\to\#} g(x) \neq 0$ *and* $g(x) \neq 0$

Examples:

1. $\lim_{x\to c} 7$ = __________
2. $\lim_{x\to c} x$ = __________
3. $\lim_{x\to 3}(x+4)$ = __________
4. $\lim_{x\to 2}(x-5)$= ___________
5. $\lim_{x\to 3}(-5x)$= __________
6. $\lim_{x\to 6} \frac{x}{2}$ = ___________
7. $\lim_{x\to 3}(5x^2)$ = ____________
8. $\lim_{x\to 2}(3x^4 - 5x^3 + 2x^2 - 6)$ = __________

Techniques for Evaluating Limits

Remember, if the function is continuous at the value indicated, use direct substitution.

If the function is NOT continuous at the value indicated:

Try 1) Factoring and reducing, 2) Long or synthetic division, 3) multiplying by the conjugate. 4) simplifying a complex expression, 5) using trig identities, or if all else fails, 6) sketch a graph of the function.

I. Dividing Out:

Factor and cancel where applicable. If you cannot factor it, then use synthetic or long division. Then use direct substitution on the simplified form (or quotient).

Ex 1: $\lim_{x\to 2} \frac{x^3-2x^2-x+2}{x^4-2x^3+x-2}$

Ex 2: $\lim_{x\to 4} \frac{2x^3-3x^2-23x+12}{x-4}$

II. Rationalizing Technique: Multiply by the conjugate to rationalize either the numerator **or** the denominator.

Ex 3: $\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

III. Using Algebra: Sometimes you have to use algebra and manipulate things:

Ex 4: $\lim_{x\to 2}\frac{\frac{1}{x+2}-\frac{1}{4}}{x-2}$

IV. Trig When you see a trig function, and direct substitution does not work, try using identities:

Ex 5: $\lim_{x\to 0}(sin^2 x + cos^2 x)$ = ________

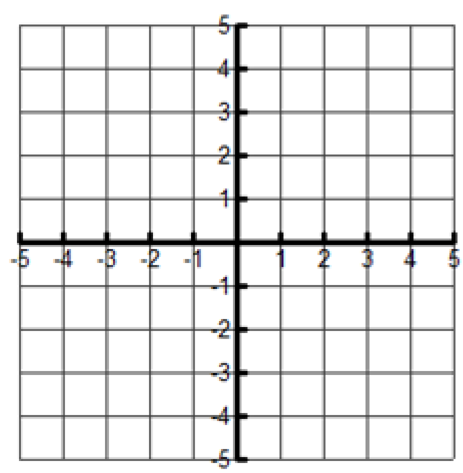
Ex 6: $\lim_{x\to 0}(\csc x \sin x)=$ __________

Ex 7: Given $f(x)=\begin{cases} 2-x & x<1 \\ 2x-x^2 & x>1 \end{cases}$, find the following:

$\lim_{x\to 1^-} f(x)=$

$\lim_{x\to 1^+} f(x)=$

$\lim_{x\to 1} f(x)=$



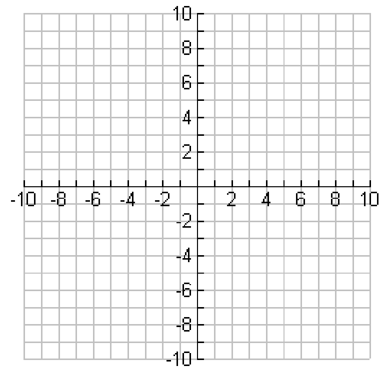
Lesson 4: Limits at Infinity

When can we do operations with infinity? (c is a constant greater than 0)

$\infty + \infty =$	$\infty + c =$
$\infty \times \infty =$	$\infty \times c =$
$\frac{\infty}{c} =$	$\frac{c}{\infty} =$
$\infty - \infty =$	$\frac{\infty}{\infty} =$

A limit is a number by definition, so in general, limits that equal ∞ or -∞ do not exist, but we can use the infinite limits to describe end behavior.

1) Graph f(x) = $\frac{3}{x-2}$. Then find each limit.



a) $\lim_{x \to 2^-} \frac{3}{x-2}$

b) $\lim_{x \to 2^+} \frac{3}{x-2}$

Vertical Asymptotes

The limit definition of a vertical asymptote is the x value where the $\lim_{x \to c} f(x) = \infty \ \text{or} \ -\infty$ from the right or the left. If the limit is a number, there is no vertical asymptote.

Determine all vertical asymptotes of the graph of each function. Justify your answer using limits.

2) $f(x) = \frac{1}{2(x+1)}$

Find each limit. (no calculator)

3) $\lim_{x\to 1}\frac{1}{x-1}$

4) $\lim_{x\to 1}\frac{1}{(x-1)^2}$

4) $\lim_{x\to 1}\frac{-1}{(x-1)^2}$

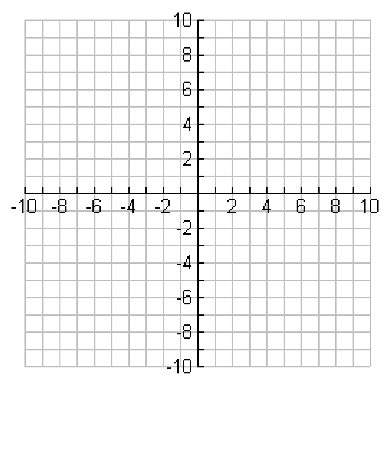
5) $\lim_{x\to 1}\frac{x^2-3x}{x-1}$

6) $\lim_{x\to 1}\frac{x^3-1}{x^2+x+1}$

Limits at Infinity

When finding a limit as $x\to\infty \text{ or } -\infty$, we only need to find the horizontal asymptote.

7) Graph $f(x)=\frac{3x^2}{x^2+1}$ then find each limit.



a) $\lim_{x\to\infty}\frac{3x^2}{x^2+1}$

b) $\lim_{x\to -\infty}\frac{3x^2}{x^2+1}$

Guideline for finding limits at $\pm\infty$ of rational functions (horizontal asymptotes)

1) If the degree of the numerator is LESS THAN the degree of the denominator, then the limit of the rational function is ___________________.

2) If the degree of the numerator is EQUAL TO the degree of the denominator, then the limit of the rational function is the _______________________________________.

3) If the degree of the numerator is GREATER THAN the degree of the denominator, then the limit of the rational function _______________________________________. (which means the limit is either ∞ or $-\infty$, and you need continue the problem to determine that)

Find each limit. (no calculator)

8) $\lim_{x\to\infty}\left(5-\frac{2}{x^2}\right)$

9) $\lim_{x\to\infty}\frac{2x-1}{x+1}$

10) $\lim_{x\to\infty}\frac{2x+5}{3x^2+1}$

11) $\lim_{x\to\infty}\sqrt{\frac{2x^2+5}{3x^2+1}}$

12) $\lim_{x\to\infty}\sqrt{\frac{2x^3+5}{3x^2+1}}$

13) $\lim_{x\to\infty}\frac{2x^2-4x}{x+1}$

14) $\lim_{x\to\infty}\frac{x^{99}}{e^x}$

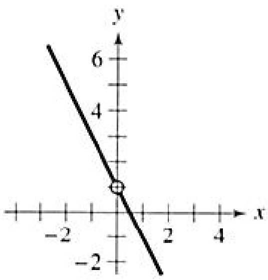
15) $\lim_{x\to\infty}\sin x$

16) $\lim_{x\to-\infty} x^3$

Limits Algebraically AND Limits to Infinity Homework

Use the graph to determine each limit (if it exists)

1. $g(x) = \dfrac{-2x^2 + x}{x}$

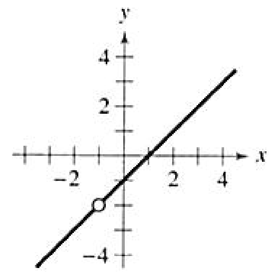


a) $\lim_{x\to 0} g(x)$

b) $\lim_{x\to -1} g(x)$

c) $\lim_{x\to -2} g(x)$

2. $f(x) = \dfrac{x^2 - 1}{x + 1}$



a) $\lim_{x\to 1} f(x)$

b) $\lim_{x\to 2} f(x)$

c) $\lim_{x\to -1} f(x)$

Find the limit (if it exists).

3. $\lim_{x\to 6} \dfrac{x-6}{x^2-36}$

4. $\lim_{x\to 2} \dfrac{x^3-8}{x-2}$

5. $\lim_{x\to 1} \dfrac{x^4-1}{x-1}$

6. $\lim_{y\to 0} \dfrac{\sqrt{5+y}-\sqrt{5}}{y}$

7. $\lim_{x\to 2} \dfrac{4-\sqrt{18-x}}{x-2}$

8. $\lim_{x\to 0} \dfrac{\dfrac{1}{x+4}-\dfrac{1}{4}}{x}$

Limits to Infinity:

9. $\lim_{x\to\infty} \frac{3}{x^2}$

10. $\lim_{x\to\infty} \frac{1-6x}{1+5x}$

11. $\lim_{x\to-\infty} \frac{e^x}{1-x^2}$

12. $\lim_{y\to\infty} \frac{4y^4}{y^2+3}$

13. $\lim_{x\to-\infty} \frac{-(x^2+3)}{(2-x)^2}$

14. $\lim_{x\to\infty} \left[7+\frac{2x^2}{(x+3)^2}\right]$

15. $\lim_{x\to\infty} \frac{3x}{1-x}$

16. $\lim_{x\to\infty} \frac{2x+1}{x^2-1}$

17. $\lim_{x\to\infty} \left(2+\frac{1}{x}\right)$

18. $\lim_{x\to\frac{\pi}{2}} \csc x$

19. $\lim_{x\to\pi} \cos x$

20. $\lim_{x\to-2} \frac{2x^3+7x^2+10x+8}{x+2}$

21. $\lim_{x\to\infty} \left(\sqrt{\frac{x^2-3x}{2x^2+5}}\right)$

22. $\lim_{x\to\infty} \left(\frac{-3x^2}{x^2+2}+\frac{5x^3}{x^4}\right)$

23-25, Find the limit from the left, from the right, and then the overall limit.

23. $\lim_{x\to6} \frac{|x-6|}{x-6}$

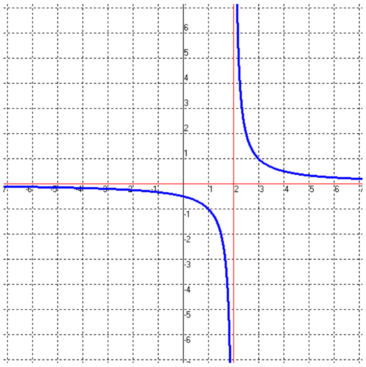
24. $\lim_{x\to1} f(x)$, where $f(x)=\begin{cases} 2x+1, & x<1 \\ 4-x^2, & x\geq1 \end{cases}$

25. $\lim_{x\to16} \frac{4-\sqrt{x}}{x-16}$

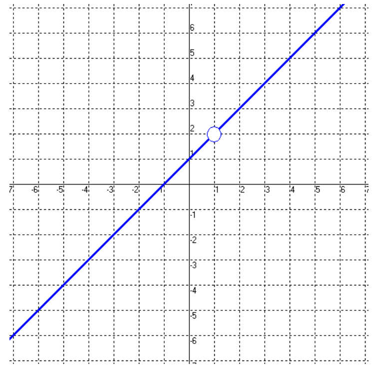
Piecewise and Limits Test Review

Use the graphs to find the limits if they exist. If not, write DNE.

1. $\lim_{x\to 2} \frac{1}{x-2}=$ $\lim_{x\to -1} \frac{1}{x-2}=$



2. $\lim_{x\to 1} \frac{x^2-1}{x-1}=$ $\lim_{x\to -5} \frac{x^2-1}{x-1}=$ $\lim_{x\to 0} \frac{x^2-1}{x-1}=$



Find the limit, if it exists.

3. $\lim_{x\to 4}\left(\frac{1}{2}x+3\right)$

4. $\lim_{x\to 3}\frac{|x-3|}{x-3}$

5. $\lim_{x\to -1}\frac{\frac{1}{x+2}-1}{x+1}$

6. $\lim_{x\to -3}\left(x^3-6x^2+3x-1\right)$

7. $\lim_{x\to 5}\frac{x-5}{x^2+5x-50}$

8. $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$

9. $\lim_{x\to \infty}\frac{4x}{2x-3}$

10. $\lim_{x\to 2} f(x)$ where $f(x)=\begin{cases}5-x, & x\le 2\\ x^2-3, & x>2\end{cases}$

11. $\lim_{x\to \infty}\frac{x^2}{2x+3}$

12. $\lim_{x\to -2}\frac{x^3-2x^2-x+14}{x+2}$

13. $\lim_{x\to \frac{3\pi}{4}} \csc x$

14. $\lim_{x\to \frac{p}{2}} \tan x$

15. $\lim_{x\to -\infty} \sin x$

16. $\lim_{x\to -\infty}\frac{x^2+3}{5x^2-4}$

17. $\lim_{x\to \infty}\sqrt{\frac{4x^2-3x+2}{5x^2-6}}$

18. $\lim_{x\to \infty}\frac{-x^2}{2x^3+3}$

19. $\lim\limits_{x \to 5} \dfrac{x^3 - 125}{x - 5}$

20. $\lim\limits_{x \to \infty} \left[\dfrac{x}{2x+1} + \dfrac{3x^2}{(x-3)^2} \right]$

21. $\lim\limits_{x \to 16} \dfrac{\sqrt{x} - 4}{x - 16}$

22. $\lim\limits_{x \to \infty} \sqrt{\dfrac{x+4}{x^2 - 6x + 3}}$

Evaluate the function for the given value of x.

$f(x) = \begin{cases} 3, if\ x\ \leq 0 \\ 2, if\ x\ > 0 \end{cases}$

$g(x) = \begin{cases} x + 5, if\ x\ \leq 3 \\ 2x - 1, if\ x\ > 3 \end{cases}$

$h(x) = \begin{cases} \frac{1}{2}x - 4, if\ x\ \leq -2 \\ 3 - 2x, if\ x\ > -2 \end{cases}$

23. f(0)

24. f($\frac{1}{2}$)

25. g(-1)

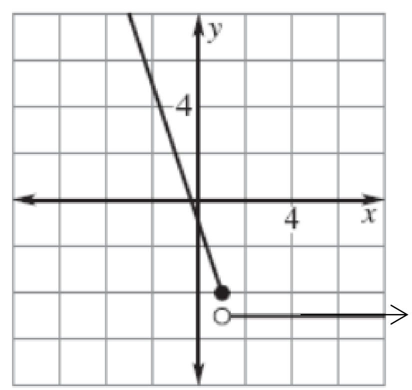
26. g(3)

27. h(-2)

28. h(6)

29. Write a piecewise function for the following graph.

(notice increments on the axis)



Graph the following.

30. f(x) = $\begin{cases} x + 3, if\ x\ \leq 0 \\ 2x, if\ x\ > 0 \end{cases}$

31. f(x) = $\begin{cases} x + 1, if\ x\ < 0 \\ -x + 1, if\ 0\ \leq x\ \leq 2 \\ x - 1, if\ x\ > 2 \end{cases}$