Notes 8.3 Geometric Sequences and Series

multiply by a constant

A sequence is **geometric** if the ratios of consecutive terms are the same. So, the sequence is geometric $a_1, a_2, a_3, \ldots a_n \ldots$ is geometric if there is a number r such that

$$\frac{a_2}{a_1} = r, \frac{a_3}{a_2} = r, \frac{a_4}{a_3} = r, \ldots$$

And so the number r is the **common ratio** of the sequence. The nth terms of these sequences are **exponential equations**.

For example,

2, 4, 8, 16, ...2^n r = 2

12, 36, 108, 324, ... $4(3^n)$ r= 3

1) Is the sequence whose n^{th} term is n^2 geometric? No - it's quadratic, not exponential

The n^{th} term of a geometric sequence has the form:

$a_n = a_1 r^{n-1}$ **or** $a_n = a_0 r^n$

where r is the common ratio of consecutive terms of the sequence.

2) Write the first five terms of the geometric sequence whose first term is $a_1 = 3$ and whose common ratio is r= 2.

3, 6, 12, 24, 48

What is the nth term of this sequence? $a_n = 3(2)^{n-1}$

3) Find the 15^{th} term whose first term is $\overset{a_1}{20}$ and whose common ratio is $\overset{r}{1.05}$.

$a_n = 20(1.05)^{n-1} \rightarrow a_{15} = 20(1.05)^{15-1} = \boxed{39.598}$

4) Find the ninth term of the geometric sequence whose first term is $\overset{a_1}{4}$ and whose common ration is $\overset{r}{½}$.

$a_n = 4(\frac{1}{2})^{n-1}$

$\rightarrow a_9 = 4(\frac{1}{2})^{9-1} = \boxed{\frac{1}{64}}$

5) Find the 12^{th} term of the sequence 5, 15, 45, ...

*3 *3

$a_1 = 5$

$r = 3$ $a_n = 5(3)^{n-1}$

$a_{12} = 5(3)^{12-1} = \boxed{885,735}$

6) Find the 10^{th} term of the sequence $\overset{a_1}{6}, \underbrace{-2}_{*-\frac{1}{3}}, \underbrace{2/3}_{*-\frac{1}{3}}, \ldots$

$a_n = 6\left(-\frac{1}{3}\right)^{n-1}$

$a_{10} = 6\left(-\frac{1}{3}\right)^{10-1} = \boxed{\dfrac{-2}{6561}}$

7) The fourth term of a geometric sequence is 125 and the 10^{th} term is $\frac{125}{64}$. Find the 14^{th} term. (Assume that the terms of the sequence are positive)

1000, 500, 250

n	a_n
4	125
10	125/64

$+6$, $*2$, $* 1/64$

$\sqrt[6]{r^6} = \sqrt[6]{\frac{1}{64}}$

$r = \frac{1}{2}$

$a_1 = 1000$

$a_n = 1000\left(\frac{1}{2}\right)^{n-1}$

$a_{14} = 1000\left(\frac{1}{2}\right)^{14-1} = \boxed{\dfrac{125}{1024}}$

8) The second term of a geometric sequence is -18 and the fifth term is 2/3. Find the sixth term.

n	a_n
1	54
2	-18
5	2/3

$+3$, $*-3$, $*\frac{-1}{27}$

$\sqrt[3]{r^3} = \sqrt[3]{-\frac{1}{27}}$

$r = -\frac{1}{3}$

$a_n = 54\left(-\frac{1}{3}\right)^{n-1}$

$a_6 = 54\left(-\frac{1}{3}\right)^{6-1} = \boxed{\dfrac{-2}{9}}$

The sum of a finite geometric sequence

...is defined by:

$$S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$$

9) Find the sum $\sum_{i=1}^{12} 4(0.3)^{i-1}$ using the formula, and then check it with your calculator.

$n=12, a_1 = 4, r = 0.3$ $S_n = 4\left(\frac{1-0.3^{12}}{1-0.3}\right) = \boxed{5.714}$

10) Find the sum $\sum_{i=1}^{7} 2^{n-1}$ using the formula, and then check it with your calculator.

$n = 7, a_1 = 1, r = 2$

$S_n = 1\left(\frac{1-2^7}{1-2}\right) = \boxed{127}$

The sum of an **infinite** geometric series...

Is shown as $S = \sum_{i=0}^{\infty} a_1 r^i = \frac{a_1}{1-r}$ ← $|r| < 1$ → sum converges to an asymptote

If |r| > 1, the sum DOES NOT EXIST!! It diverges.

11) Find the sum $\sum_{n=1}^{\infty} 4(0.6)^{n-1}$

$a_1 = 4, r = 0.6$ $S_n = \frac{4}{1-0.6} = \boxed{10}$

12) Find the sum of $3 + \underbrace{0.3}_{*\frac{1}{10}} + \underbrace{0.03}_{*\frac{1}{10}} + 0.003 + \ldots$ (Give an exact answer)

$a_1 = 3$
$r = \frac{1}{10}$

$S_n = \frac{3}{1-1/10} = \boxed{\dfrac{10}{3}}$

13) Find the sum: $3 + \underbrace{6}_{*2} + \underbrace{12}_{*2} + 24 + \ldots$

$a_1 = 3$
$r = 2$

$r > 1 \rightarrow \boxed{\text{Sum DNE}}$

~~14) Write $0.\overline{18}$ as a rational number using a geometric series.~~

15) A deposit of \$50 is made on the first day of each month in a savings account that pays 6% compounded monthly. What is the balance at the end of two years? (This is called an increasing annuity)

$a_n = 50(1.06)^{n-1}$ $n = 24$ (24 months in 2 years)

$\sum_{n=1}^{24} 50(1.06)^{n-1} = \boxed{\$2540.78}$