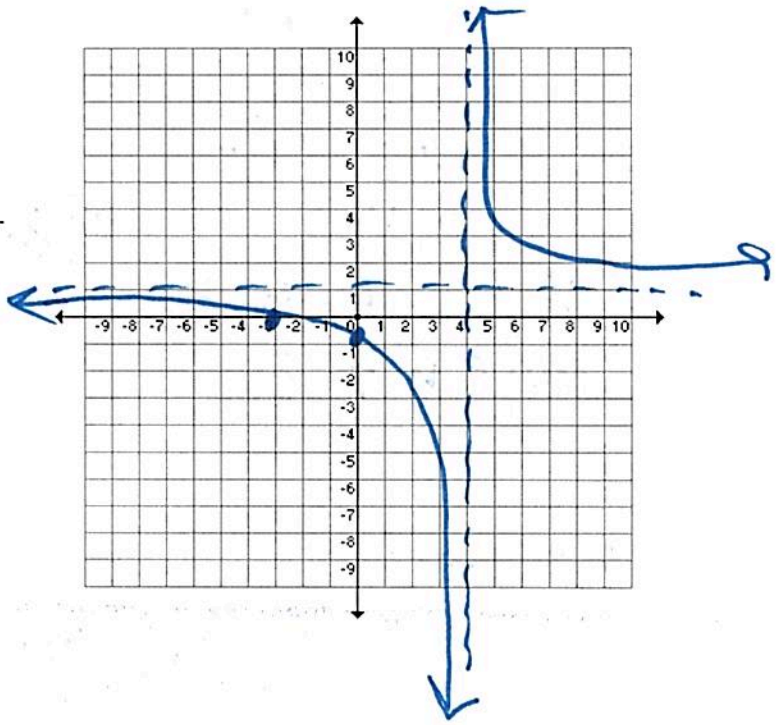
Pre-AP Precalculus

Name Key

Rational Functions Practice

Graph the following rational functions. Be sure to label asymptotes and holes in the graph, and find the specified limits for the function.

1) $f(x) = \dfrac{x+3}{x-4}$



Horizontal asymptote: $y = 1$

Removable discontinuity: none

Vertical asymptote: $x = 4$

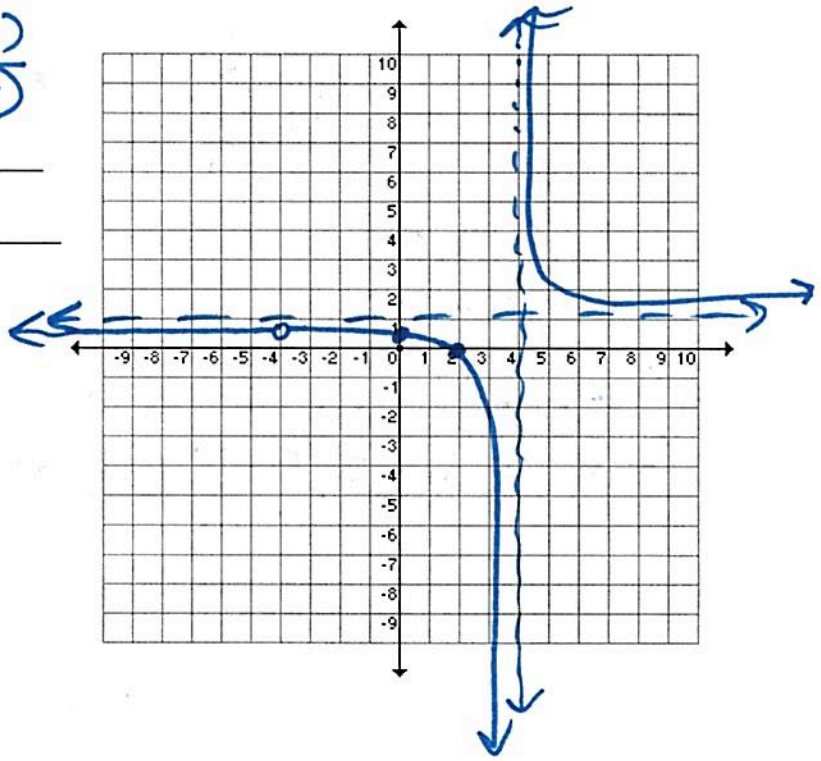
x-intercept: $(-3, 0)$

y-intercept: $(0, -3/4)$

domain: $\mathbb{R}$, $x \neq 4$

range: $\mathbb{R}$, $y \neq 1$

2) $f(x) = \dfrac{x^2+2x-8}{x^2-16} = \dfrac{(x+4)(x-2)}{(x+4)(x-4)}$



Horizontal asymptote: $y = 1$

Removable discontinuity: $(-4, 3/4)$

Vertical asymptote: $x = 4$

x-intercept: $(2, 0)$

y-intercept: $(0, 1/2)$

domain: $\mathbb{R}$, $x \neq 4, -4$

range: $\mathbb{R}$, $y \neq 1, 3/4$

$3(x^2+x-2)$

3) $f(x)=\dfrac{3x^2+3x-6}{x^2+x-12}$ = $\dfrac{3(x+2)(x-1)}{(x+4)(x-3)}$

Horizontal asymptote: $y=3$

Removable discontinuity: none

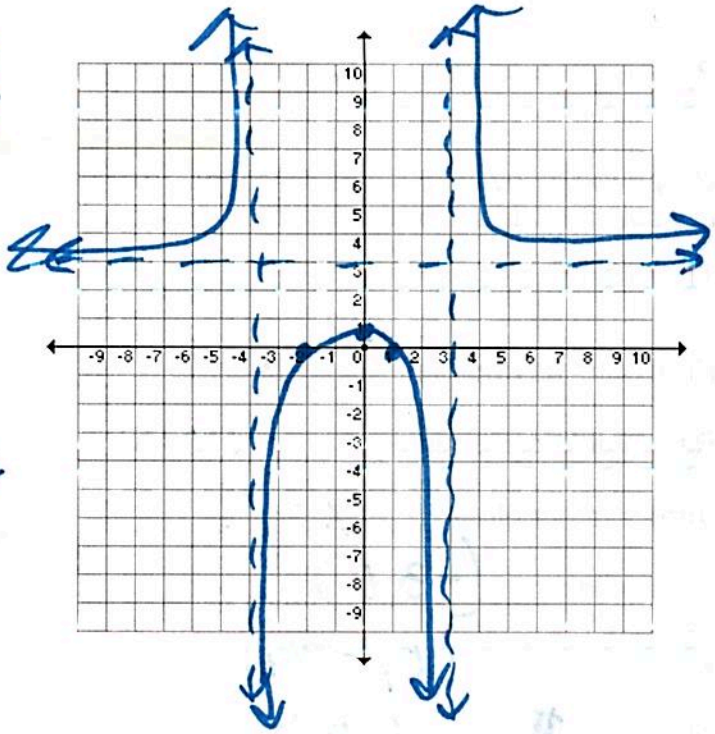
Vertical asymptote: $x=-4,\ x=3$

x-intercept: $(-2,0)$ $(1,0)$

y-intercept: $(0,\ 1/2)$ $-6/-12$

domain: $\mathbb{R},\ x\neq -4, 3$

range: can't tell



$2(x^2+3x-4)$

4) $f(x)=\dfrac{2x^2+6x-8}{x^2-1}$ = $\dfrac{2(x+4)(x-1)}{(x-1)(x+1)}$

Horizontal asymptote: $y=2$

Slant asymptote: none

Removable discontinuity: $(1,5)$

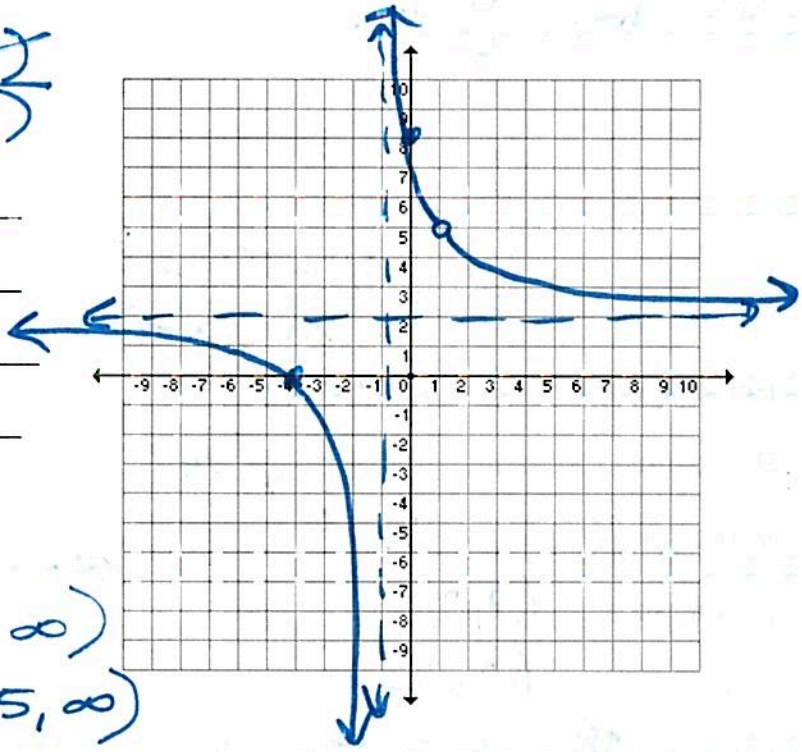
Vertical asymptote: $x=-1$

x-intercept: $(-4,0)$

y-intercept: $(0,8)$

domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

range: $(-\infty,2)\cup(2,5)\cup(5,\infty)$



Does this graph cross the horizontal asymptote, and if so where? no

$$2=\frac{2(x+4)}{x+1}$$

$$2(x+1)=2(x+4)$$

5) $f(x) = \dfrac{3x^2 - 12x - 15}{(x^3 - 2x^2) - (25x + 50)} = \dfrac{3(x-5)(x+1)}{(x-5)(x+5)(x-2)}$

$3(x^2-4x-5)$

$x^2(x-2) - 25(x-2)$

Horizontal asymptote: $y = 0$

Slant asymptote: none

Removable discontinuity: $(5, 3/5)$

Vertical asymptote: $x = -5, x = 2$

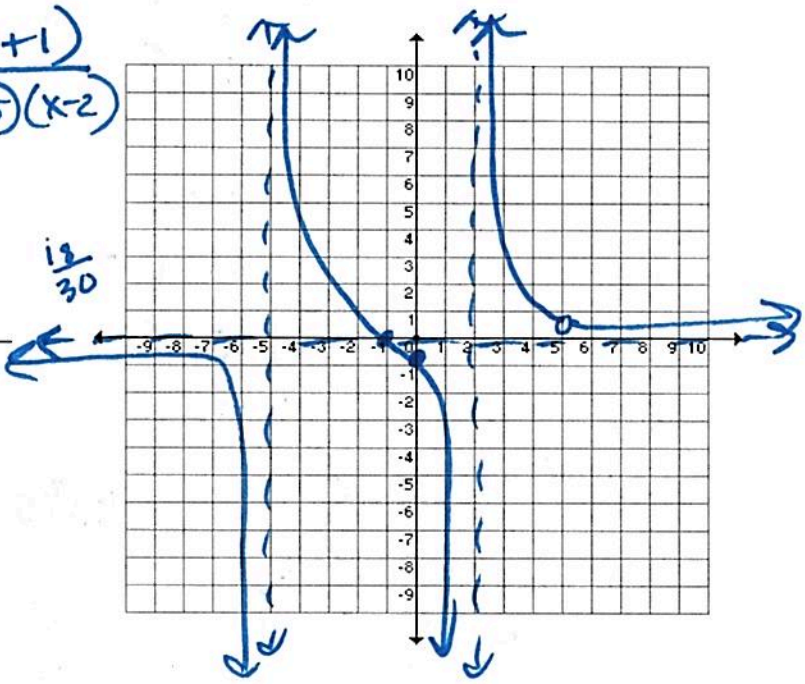
x-intercept: $(-1, 0)$

y-intercept: $(0, -3/10)$

domain: $\mathbb{R}, x \neq -5, 2, 5$

range: $\mathbb{R}$

$13/30$



Does this graph cross the horizontal asymptote, and if so where? yes @ $x = -1$

6) $f(x) = \dfrac{3x^3 - 12x}{(x^3 - 2x^2) - (16x + 32)} = \dfrac{3x(x-2)(x+2)}{(x-2)(x+4)(x-4)}$

$3x(x^2-4)$

$x^2(x-2) - 16(x-2)$

Horizontal asymptote: $y = 3$

Slant asymptote: none

Removable discontinuity: $(2, -2)$

Vertical asymptote: $x = 4, x = -4$

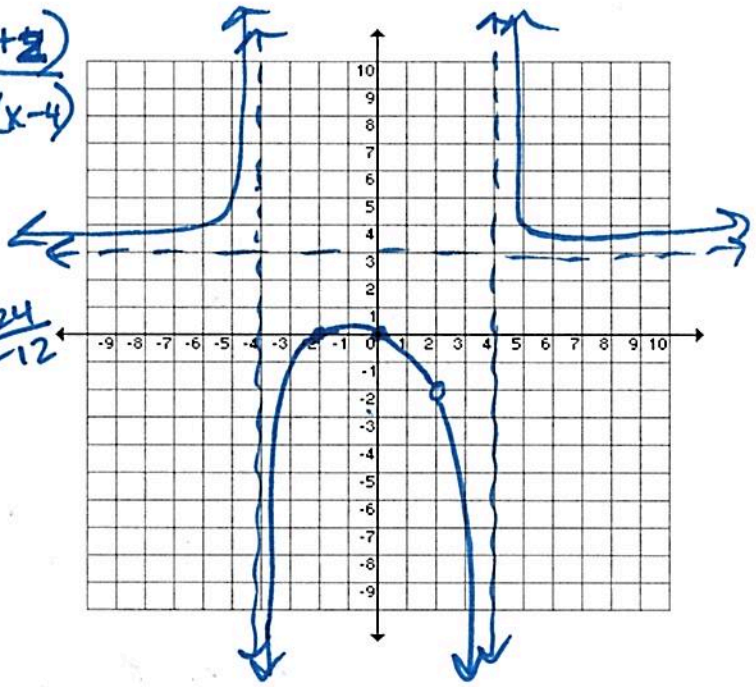
x-intercept: $(-2, 0)$ $(0, 0)$

y-intercept: $(0, 0)$

domain: $\mathbb{R}, x \neq 4, -4, 2$

range: can't tell

$24/-12$

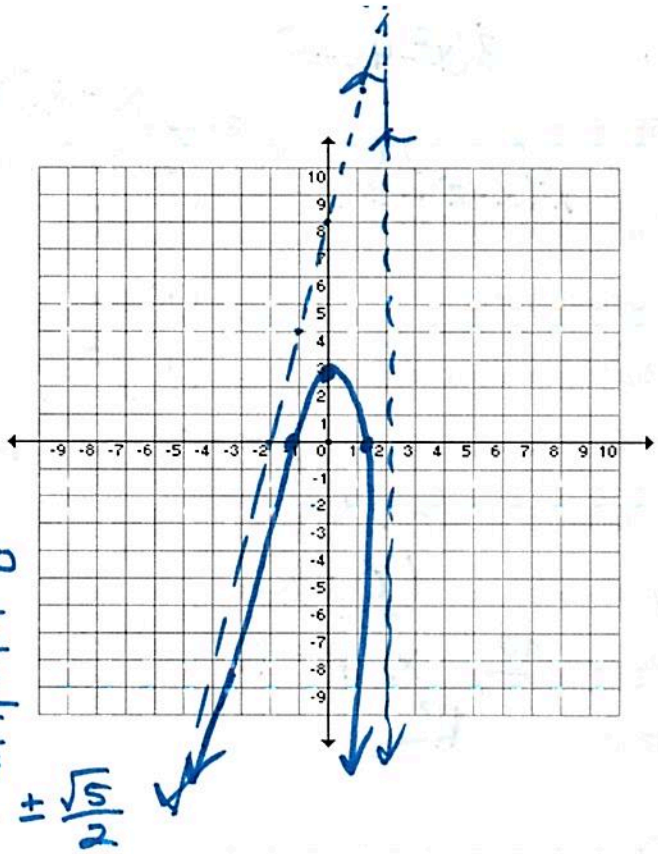


Does this graph cross the horizontal asymptote, and if so where? no

$3(x^2-16) \neq 3x^2+16x$

7) $g(x) = \frac{4x^2 - 5}{x - 2}$

2 | 4 0 −5
↓ 8 16
4 8 x



Horizontal asymptote: none

Slant asymptote: $y = 4x + 8$

Removable discontinuity: none

Vertical asymptote: $x = 2$

x-intercept: $(\pm\sqrt{5}/2, 0)$

y-intercept: $(0, 5/2)$

domain: $(-\infty, 2) \cup (2, \infty)$

range: can't tell

$4x^2 - 5 = 0$

$\frac{4x^2}{4} = \frac{5}{4}$

$x^2 = \sqrt{\frac{5}{4}}$

$x = \pm\frac{\sqrt{5}}{2}$

~~8) Find the vertical and non-linear asymptotes for the following function.~~

~~$\frac{x^3 - 3x^2 + 1}{x - 2}$~~

For the following questions, create your own function that will meet the given criteria. Leave your answer in factored form please! Note: there are a variety of correct answers.

9) A function that has a vertical asymptote at x = 3.

$y = \frac{x}{x-3}$

10) A function that has a vertical asymptote at x = 5 and a hole at x = 1.

$y = \frac{(x-1)}{(x-1)(x-5)}$

11) A function has a zero at x = 3, a vertical asymptote at x = -4 and a hole at x = 0.

$y = \frac{x(x-3)}{x(x+4)}$

12) A function that has a vertical asymptote at x = 5, a hole at x = -2, and a horizontal asymptote at y = 0.

$y = \frac{(x+2)}{(x+2)(x-5)}$