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VECTORS

The **component form** of the vector with initial point P = (x_1, y_1) and terminal point Q = (x_2, y_2) is given by

$$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle = v$$

Sometimes this form is also called a **position** vector.

Its **magnitude** (or length) is denoted by $\|\overrightarrow{PQ}\|$ and can be found using the distance formula or Pythagorean Theorem.

$$\|\overrightarrow{PQ}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

or Pythagorean Theorem ☆

1) Find the component form and magnitude of the vector **v** that has initial point (-2, 3) and terminal point (-7, 9).

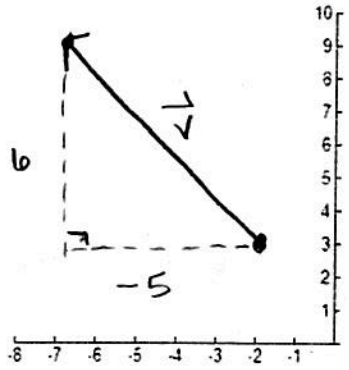
$x_1\ y_1$ $x_2\ y_2$

$\vec{v} = \langle -7+(-2), 9-3 \rangle$

$\vec{v} = \langle -5, 6 \rangle$

$\|\vec{v}\| = \sqrt{(-5)^2 + 6^2}$

$\|\vec{v}\| = \sqrt{61}$



Vector addition: algebraically use component forms.

Let $\mathbf{u} = \langle 1, 2 \rangle$ and $\mathbf{v} = \langle 3, 1 \rangle$. Find each of the following vectors.

2) $\mathbf{u} + \mathbf{v}$

$\langle 1,2 \rangle + \langle 3,1 \rangle$ (+ +)

$= \langle 4, 3 \rangle$

3) $\mathbf{u} - \mathbf{v}$

$\langle 1,2 \rangle - \langle 3,1 \rangle$ (− −)

$= \langle -2, 1 \rangle$

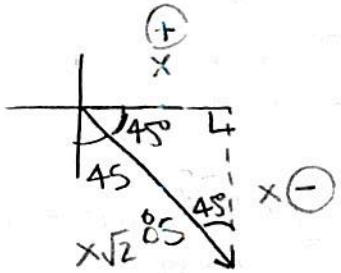
4) $2\mathbf{u} - 3\mathbf{v}$

$2\langle 1,2 \rangle - 3\langle 3,1 \rangle$

$\langle 2,4 \rangle - \langle 9,3 \rangle$ (− −)

$= \langle -7, 1 \rangle$

5) A car travels S 45° E for 85 miles. Determine the vector written in component form that models the car's trip.



$\left\langle \frac{85}{\sqrt{2}}, -\frac{85}{\sqrt{2}} \right\rangle$

$\frac{x\sqrt{2}}{\sqrt{2}} = \frac{85}{\sqrt{2}} \rightarrow x = \frac{85}{\sqrt{2}}$

There are two standard unit vectors: $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.

In the vector $\langle v_1, v_2 \rangle$, v_1 represents the horizontal component and v_2 the vertical component.

$v_1\mathbf{i} + v_2\mathbf{j}$ is called a linear combination of the vectors **i** and **j**.

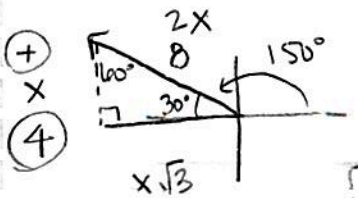
Example: $\mathbf{v} = \langle 5, 2 \rangle$ can be written as $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j}$.

x y

6) Write the vector $\mathbf{v} = 3\mathbf{i} - 8\mathbf{j}$ in component form.

$\langle 3, -8 \rangle$

7) Given the magnitude of vector **v** is 8 and the angle that it makes with the positive x-axis is 150°. Write a vector as a linear combination in the form $a\mathbf{i} + b\mathbf{j}$.



$2x = 8$

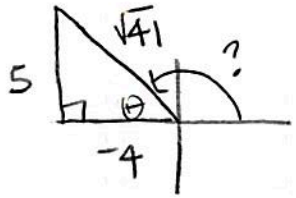
$x = 4$

$\vec{v} = -4\sqrt{3}\,i + 4j$

8) Let $v = -4i + 5j$. Find the **direction angle** for vector v measured from the positive x-axis.

$\vec{v} = -4i + 5j$

$\vec{v} = \langle -4, 5 \rangle$



$\tan\theta = \frac{5}{-4}$

$\theta = \tan^{-1}\left(\frac{5}{-4}\right)$

$\theta = -51.340 \rightarrow 51.340°$

$180° - 51.340°$

$= 128.166°$