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Notes 2.4: Complex Numbers

All complex numbers can be written in the form **a ± bi** form (called standard form) where a is the real part and bi is the imaginary part.

<u>If b = 0, the number is a real number</u>.

Examples of real numbers: 4, -5, $\sqrt{24}$, $\frac{3}{5}$, etc.

<u>If a = 0 it is called a pure imaginary number</u>.

Examples 5i , -8i, etc.

<u>Oftentimes, complex numbers will have a real part (the "a" part) and an imaginary part (the "bi" part).</u>

Examples: -5 + 3i, 8 – 5i, etc.

The Imaginary Unit i

$i = \sqrt{-1}$, therefore, $i^2 =$ -1

$\sqrt{-9} = (\sqrt{9})(\sqrt{-1}) = 3i$
$\quad\quad\;\; 3 \quad\;\; i$

Simplify:

1) i^4
$(i^2)^2$
$(-1)^2 = \boxed{1}$

2) i^{25}
$i(i^2)^{12}$
$i(-1)^{12}$
$i(1) = \boxed{i}$

3) i^{64}
$(i^2)^{32}$
$(-1)^{32} = \boxed{1}$

4) i^{35}
$i(i^2)^{17}$
$i(-1)^{17}$
$i(-1) = \boxed{-i}$

Sometimes complex numbers can be simplified.

5) $-8 + \sqrt{-25}$
$\sqrt{-1} \cdot \sqrt{25}$
$i \quad 5$
$\boxed{-8 + i\sqrt{5}}$

6) $17x - \sqrt{-18}$
$\sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{2}$
$i \quad 3$
$\boxed{17x - 3i\sqrt{2}}$

7) $\sqrt{-63}$
$\sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{7}$
$i \quad 3$
$\boxed{3i\sqrt{7}}$

8) $\sqrt{-81x}$
$\sqrt{-1} \cdot \sqrt{81} \cdot \sqrt{x}$
$i \quad 9$
$\boxed{9i\sqrt{x}}$

When we add and subtract complex numbers, we add/subtract the real part together and the imaginary part together. When we multiply complex numbers, you will need to distribute and simplify.

9) $(4 + 7i) + (1 - 6i)$

$\boxed{5+i}$

10) $4(-2 + 3i)$

$\boxed{-8+12i}$

11) $(1 + 2i) - (4 + 2i)$

$1+\cancel{2i}-4-\cancel{2i}$

$\boxed{-3}$

12) $(2 - i)(4 + 3i)$

$8+6i-4i-3\cancel{i^2}(-1)$ $\quad -3(-1)$

$8+2i+3$

$\boxed{11+2i}$

<u>Complex Conjugates</u>

For any complex # (a + bi), its conjugate is (a – bi)

$(a + bi)(a - bi) \quad = a^2 - abi + abi + b^2i^2$

$= a^2 - b^2(-1)$

$= a^2 + b^2$

For example:

13) $(3 + 2i)(3 - 2i)$

$9-\cancel{6i}+\cancel{6i}-4\cancel{i^2}(-1)$

$-4(-1)$

$9+4$

$=\boxed{13}$

14) $(1 + i)$ multiplied by its conjugate

$(1+i)(1-i)$

$1-\cancel{i}+\cancel{i}-\cancel{i^2}(-1)$

$-(-1)$

$1+1$

$=\boxed{2}$

When we divide complex numbers (or write a quotient), we are not allowed to leave i's in the denominator. <u>To fix this, multiply both the numerator and denominator by the complex conjugate.</u>

15) $\dfrac{2 + 3i}{4 - 2i} \cdot \dfrac{(4+2i)}{(4+2i)}$

$\dfrac{8+4i+12i+6\cancel{i^2}(-1)}{16-4\cancel{i^2}(-1)}$ $\quad -6 \quad -4(-1)$

$\dfrac{2+16i}{20} \rightarrow \boxed{\dfrac{1+8i}{10}}$

16) $-\dfrac{14}{2i} \cdot \dfrac{2i}{2i} = \dfrac{-28i}{4\cancel{i^2}(-1)}$

$\dfrac{-28i}{-4} =$

$\boxed{7i}$

<u>Complex Solutions of Quadratics</u>

Solve the following quadratics that have complex solutions:

17) $x^2 + 4 = 0$

$\sqrt{x^2} = \pm\sqrt{-4} < \begin{matrix}\sqrt{-1} & i \\ \sqrt{4} & 2\end{matrix}$

$\boxed{x = \pm 2i}$

18) $3x^2 - 2x + 5 = 0$

Not factorable!

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)}$ $\quad 4-60$

$x = \dfrac{2 \pm \sqrt{-56}}{6} < \begin{matrix}\sqrt{-1} & i \\ \sqrt{4} & 2 \\ \sqrt{14}\end{matrix}$

$x = \dfrac{2 \pm 2i\sqrt{14}}{6}$

$\boxed{x = \dfrac{1 \pm i\sqrt{14}}{3}}$