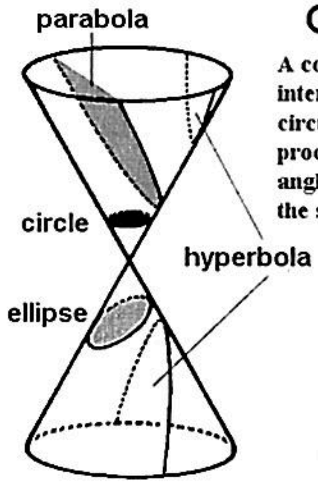
NOTES ON CONIC SECTIONS DAY 1:
WHAT IS A CONIC SECTION?

What is a conic section?



Conic Sections

A conic section is formed by the intersection of a plane with a right circular cone. The "kind" of curve produced is determined by the angle at which the plane intersects the surface.

Classifying Conics:

Any conic section can be described by a general second-degree equation in x and y:
$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$

If A and C are the same, then you have a circle.
If A and C are opposite signs, then you have a hyperbola.
If A and C are the same sign, but different numbers, then you have an ellipse.
If either A or C is 0, then you have a parabola.

Examples:

$-x^2 + 10x + y - 21 = 0$

parabola

$-2y^2 + x - 20y - 49 = 0$

parabola

$x^2 + 2x + y - 1 = 0$

parabola

$x^2 + y^2 + 6x - 2y + 9 = 0$

circle

$x^2 - y^2 - 2x - 8 = 0$

hyperbola

$3x^2 + 30x + y^2 + 79 = 0$

ellipse

$-9x^2 + y^2 - 72x - 153 = 0$

hyperbola

$-8x^2 - 2x - 4y^2 - 100 = 0$

ellipse

circle – both $x^2 + y^2$, same sign, same coefficient
parabola – only x^2 or y^2, not both
ellipse – both $x^2 + y^2$, same sign, different coeff.
hyperbola – both $x^2 + y^2$, different signs

Circles:

A circle is the set of all points equidistant from a given point (the center).

The equation of a circle with a center at (h, k) and a radius of r is $(x-h)^2+(y-k)^2=r^2$

Given the equation, identify the center and radius of each circle. Then graph.

1. $(x+3)^2+(y+1)^2=4$

Center $(-3,-1)$ Radius 2

2. $5x^2+5y^2-125=0 \rightarrow \frac{5x^2+5y^2}{5}=\frac{125}{5}$

Center $(0,0)$ Radius 5

$x^2+y^2=25$

Given the center and the radius, write the equation of the circle.

$(2\sqrt{5})(2\sqrt{5})$

$4(5)=20$

3. Center (4, 6), r = 11

$(x-4)^2+(y-6)^2=121$

4. Center (-8, -5), r = $\sqrt{15}$

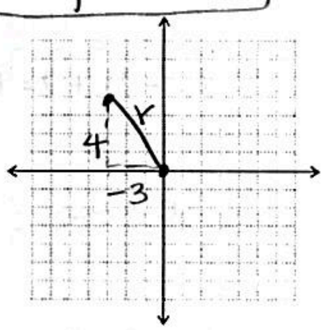
$(x+8)^2+(y+5)^2=15$

5. Center (0, 0), r = $2\sqrt{5}$

$x^2+y^2=20$

6. Write the equation of a circle with a center at (0, 0) and a solution point at (-3,

4). $x^2+y^2=25$



$r=\sqrt{(-3)^2+4^2}$

$=\sqrt{25}=5$

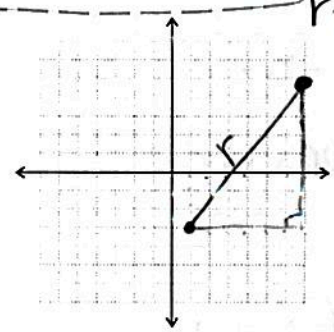
7. Write the equation of a circle with a center at (1, -3) and a solution point at (7,

5). $(x-1)^2+(y+3)^2=100$

$r=\sqrt{6^2+8^2}$

$r=\sqrt{100}$

$r=10$



Write the equation in standard form and identify the center and radius.

8. $x^2+y^2+16x+4y+2=0$

$(x^2+\underset{\frac{16}{2}}{16x}+64)+(y^2+\underset{\frac{4}{2}}{4y}+4)=-2+64+4$

$(8)^2 \quad (2)^2$

$(x+8)^2+(y+2)^2=66$

Center $(-8,-2)$ Radius $\sqrt{66}$

9. $x^2+y^2+18x-6y-31=0$

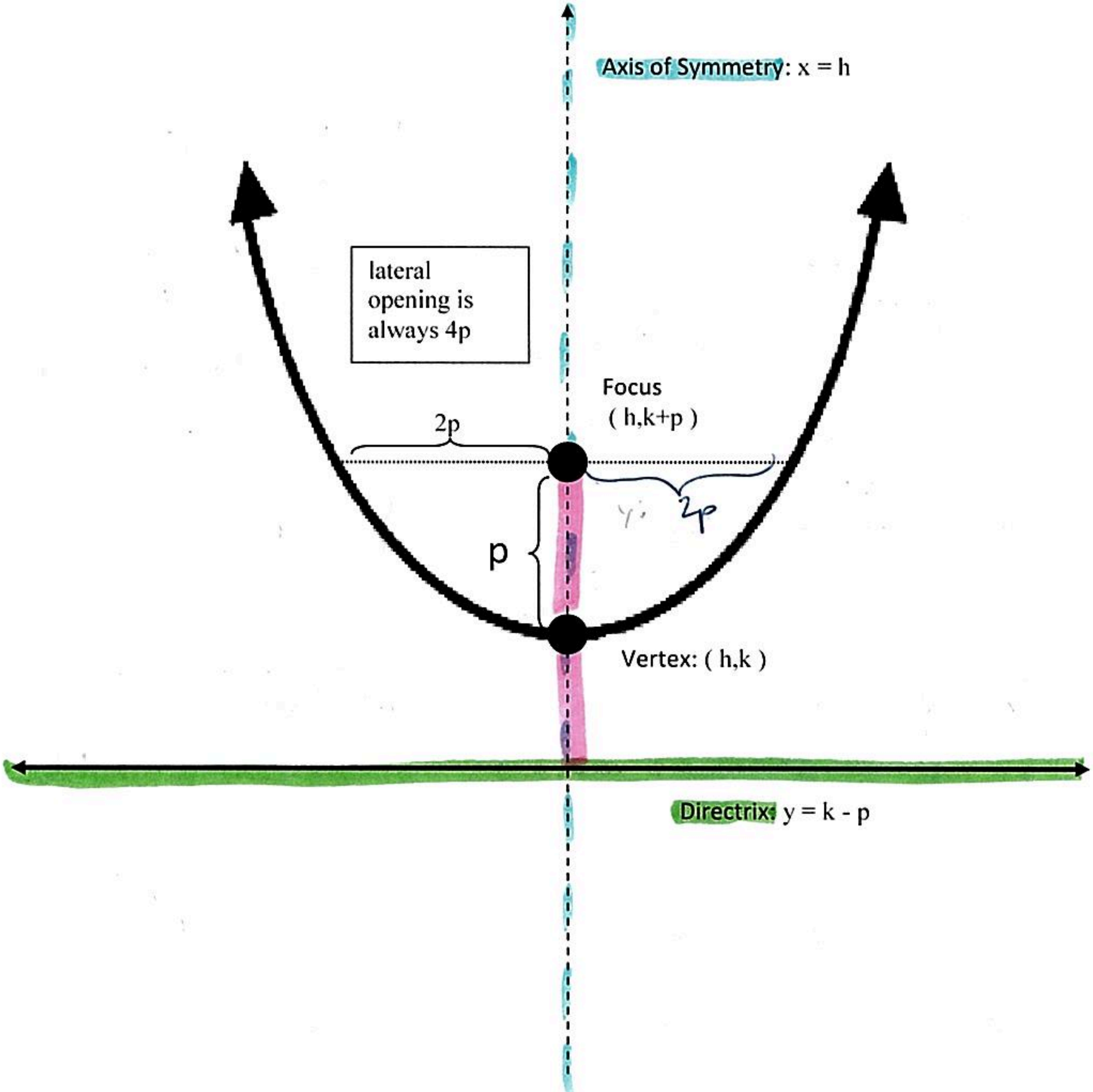
$(x^2+\underset{\frac{18}{2}}{18x}+81)+(y^2-\underset{\frac{6}{2}}{6y}+9)=31+81+9$

$(9)^2 \quad (-3)^2$

$(x+9)^2+(y-3)^2=121$

Center $(-9,3)$ Radius 11

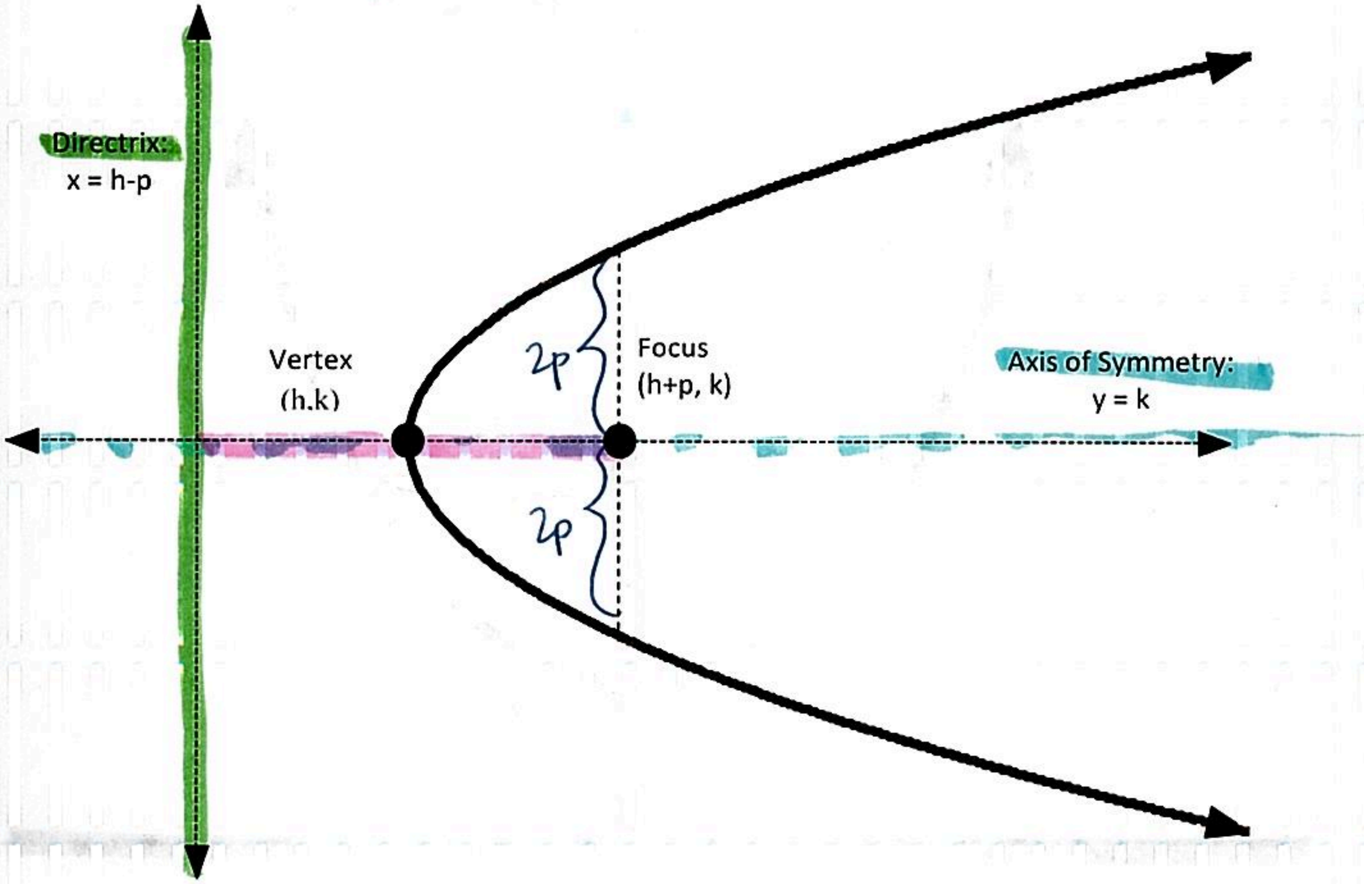
Vertical Parabola



	Vertex Form	Conic Standard Form
Vertical	$\mathbf{y = a(x-h)^2+k}$	$\mathbf{(x-h)^2 = 4p(y-k)}$
Horizontal	$x = a(y-k)^2+h$	$(y-k)^2=4p(x-h)$

****$a = \frac{1}{4p}$***

Horizontal Parabola

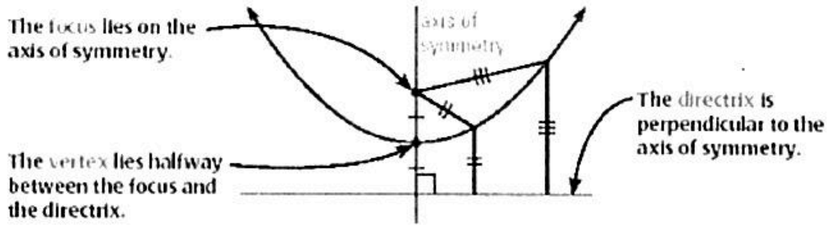


	Vertex Form	Conic Standard Form
Vertical	$y = a(x-h)^2+k$	$(x-h)^2 = 4p(y-k)$
Horizontal	$\mathbf{x = a(y-k)^2+h}$	$\mathbf{(y-k)^2=4p(x-h)}$

$****a = \frac{1}{4p}***$

Parabolas:

A parabola is the set of all points P(x, y) whose distance from the focus (F) equals its distance from the directrix.



EQUATION	VERTEX	FOCUS	DIRECTRIX	AXIS OF SYMMETRY
$(y-k)^2 = 4p(x-h)$	(h, k)	(h + p, k)	x = h – p	HORIZONTAL y=k
$(x-h)^2 = 4p(y-k)$	(h, k)	(h, k + p)	y= k – p	VERTICAL x = h

Examples:

1. $(x-1)^2 = 4(y+2)$

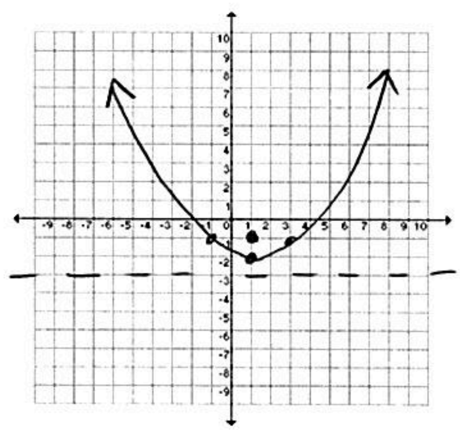
Vertex $(1,-2)$

Directrix: $y=-3$

Focus: $(1,-1)$

h = 1
k = −2
4p= 4
p= 1
~~lateral opening is~~ 4

direction of opening =



2. $(y+1)^2 = -8(x-2)$

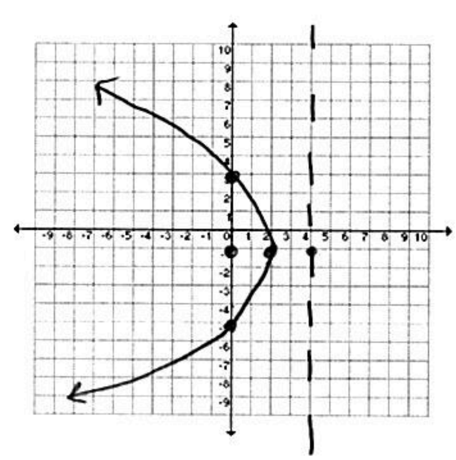
Vertex: $(2,-1)$

Directrix: $x=4$

Focus: $(0,-1)$

h = 2
k = −1
4p= 8 4p=8
p= 2
~~lateral opening is~~ 8

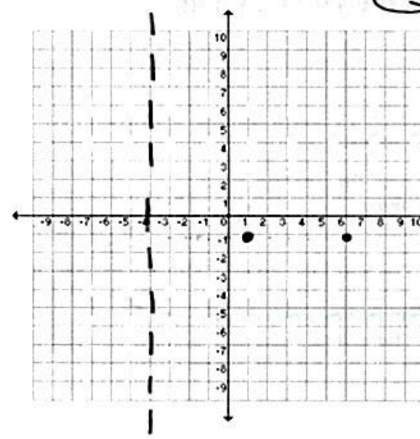
direction of opening =



Use the graph and the given information to find the equation of the parabola.

3. Directrix: x = - 4, Vertex (1, -1)

$$\boxed{(y+1)^2 = 20(x-1)}$$



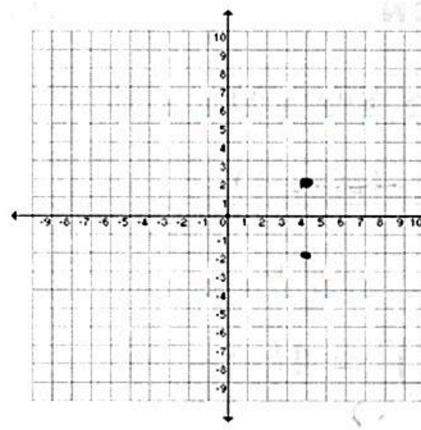
$+y^2$

$p=5$

$4p=20$

4. Vertex (4, 2), Focus at (4, -2).

$$\boxed{(x-4)^2 = -16(y-2)}$$



$-x^2$

$p=4$

$4p=16$