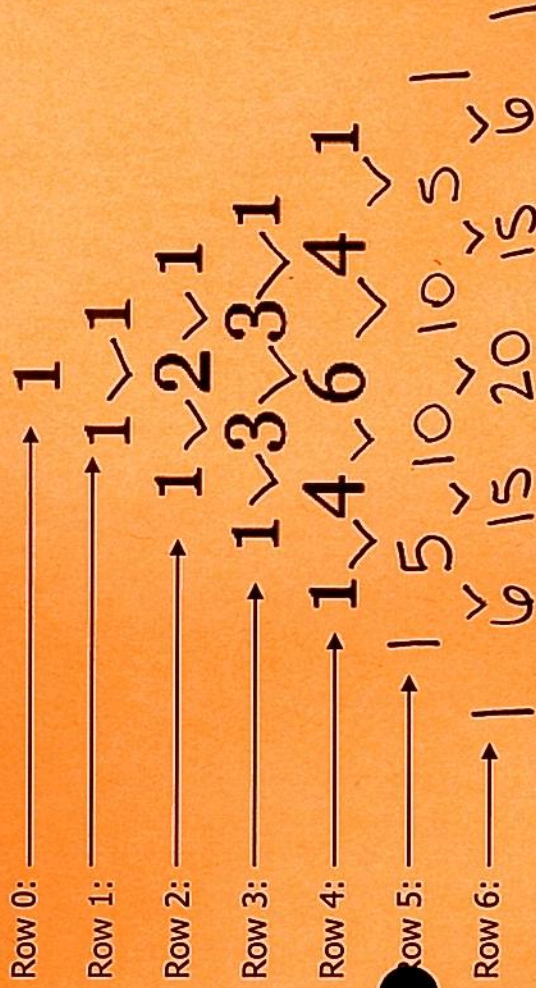


### The Binomial Theorem

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

1. There will be  $n + 1$  terms
2.  $x$  and  $y$  have symmetrical roles
3. the sum of the powers of each term is  $n$ . (as long as  $x$  and  $y$  are not to a power)
4. coefficients increase then decrease in a symmetrical pattern

Pascal's triangle tells you the coefficients for your expanded binomial:



# Binomial Theorem

① Expand  $(x+y)^6$  — use row 6

Coeff.	1st	2nd	Term
1	$x^6$	$y^0 = 1$	$x^6$
6	$x^5$	$y^1 = y$	$6x^5y$
15	$x^4$	$y^2$	$15x^4y^2$
20	$x^3$	$y^3$	$20x^3y^3$
15	$x^2$	$y^4$	$15x^2y^4$
6	$x^1 = x$	$y^5$	$6xy^5$
1	$x^0 = 1$	$y^6$	$y^6$

$$x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

② Expand  $(x-2y)^4$  — use row 4

Coeff.	1st	2nd	Term
1	$x^4$	$(-2y)^0 = 1$	$x^4$
4	$x^3$	$(-2y)^1 = -2y$	$-8x^3y$
6	$x^2$	$(-2y)^2 = 4y^2$	$24x^2y^2$
4	$x^1 = x$	$(-2y)^3 = -8y^3$	$-32xy^3$
1	$x^0 = 1$	$(-2y)^4 = 16y^4$	$16y^4$

$$x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$$



③  $(2x^2 - y^3)^3 \rightarrow$  Expand. use row 3

Coeff.	1st	2nd	Term
1	$(2x^2)^3 = 8x^6$	$(-y^3)^0 = 1$	$8x^6$
3	$(2x^2)^2 = 4x^4$	$(-y^3)^1 = -y^3$	$-12x^4y^3$
3	$(2x^2)^1 = 2x^2$	$(-y^3)^2 = y^6$	$6x^2y^6$
1	$(2x^2)^0 = 1$	$(-y^3)^3 = -y^9$	$-y^9$

$$8x^6 - 12x^4y^3 + 6x^2y^6 - y^9$$

Practice:

Expand the following:

①  $(x - 3y)^4$

②  $(3x + y^2)^5$

③  $(2x + 3y)^6$