Notes 8.2 Arithmetic Sequences and Partial Sums

Arithmetic Sequences

A sequence is **arithmetic** if the differences between consecutive terms is the same. constant rate of change

The n^{th} term of an arithmetic sequence has the explicit form $a_n = dn + a_0$, where d is the common difference between consecutive terms.

$d = a_2 - a_1 = a_3 - a_2$ ... etc.

n= term # a_0= zero-eth term

Therefore, arithmetic sequence can be thought of as linear functions.

Some people prefer to use the explicit form: **$a_n = a_1 + d(n-1)$** simplify down!

a_1= first term

The nature of the problem dictates which formula is faster/easier so I like to know both.

a) Find a_n for the sequence 7, 11, 15, 19....

+4 +4 +4

first: identify arithmetic, geometric, or neither. arithmetic $a_1 = 7$

second: find the common difference, or common ratio if there is one. $d = 4$

third: write explicit equation $a_n = 7 + 4(n-1)$

$7 + 4n - 4 \rightarrow \boxed{a_n = 4n + 3}$

option 1: plug into formula

$a_n = a_1 + d(n-1)$

$a_n = 7 + 4(n-1)$

$a_n = 4n + 3$

option 2: go back to the 0^{th} term

$a_n = dn + a_0$

$a_0 = a_1 - d = 7 - 4 = 3$

$a_n = 4n + 3$

b) Find a_n for the sequence 2, -3, -8, -13, ...?

-5 -5 -5

$a_1 = 2$

$d = -5$

$a_n = 2(-5)(n-1)$

$2 - 5n + 5$

$\boxed{a_n = -5n + 7}$

c) Find the formula for the n^{th} term of the arithmetic sequence whose common difference is 3 and whose first term is 2.

a_1 (2), d (3)

$a_n = 2+3(n-1)$

$2+3n-3$

$\boxed{a_n = 3n-1}$

d) The fourth term of an arithmetic sequence is 20, and the 13^{th} term is 65. Write the formula for the nth term of the sequence and give the formula for a_n.

n	a_n
1	5
2	10
3	15
4	20
13	65

$+9$ (4 to 13), $+45$

$d = \frac{45}{9} = 5$

$a_1 = 5$

$a_n = 5+5(n-1)$

$5+5n-5$

$\boxed{a_n = 5n}$

The Sum of a Finite Arithmetic Sequence

The sum of a finite arithmetic sequence with n terms is

$\sum_{i=1}^{n} a_n = S_n = \frac{n}{2}(a_1 + a_n)$ n= number of terms a_n= last term in sequence

* Need n, a_1 & a_n

f) Find the sum: 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19.

$a_1 = 1$

$a_n = 19$

$n = 10$

$S_{10} = \frac{10}{2}(1+19) = \boxed{100}$

g) Find the sum of the integers from 1 to 120.

a_1, a_n, $n = 120$

$S_{120} = \frac{120}{2}(1+120) = \boxed{7260}$

h) Write the sigma notation for the 150^{th} partial sum of the arithmetic sequence 5, 16, 27, 38, 49....., then find the sum.

upper limit

+11 +11 +11

$a_1 = 5$

$d = 11$

$a_n = 5+11(n-1)$

$5+11n-11$

$a_n = 11n-6$

$\sum_{n=1}^{150} (11n-6) = \boxed{123,675}$

i) In a golf tournament, the 16 golfers with the lowest scores win cash prizes. First place receives a cash prize of $1000, second place receives $950, third place receives $900, and so on. What is the total amount of prize money?

need $\sum$

1000, 950, 900

−50 −50

$d = -50$

$a_1 = 1000$

upper limit = 16

$a_n = 1000-50(n-1)$

$a_n = 1000-50n+50$

$a_n = -50n+1050$

$\sum_{n=1}^{16} (-50n+1050) = \boxed{\$10,000}$