PAP Precal HW: Zeros of Polynomial Functions

1) Find the zeros of $f(x) = x^3 - 4x^2 - x + 4$

Possible Rational Roots: $\pm 4 \pm 2 \pm 1$

Use synthetic division to test your possible roots:

$x = 1$

$$\begin{array}{c|cccc} 1\rfloor & 1 & -4 & -1 & 4 \\ & & 1 & -3 & -4 \\ \hline & 1 & -3 & -4 & 0 \checkmark \end{array}$$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1)(x-1)$

$x = 4 \quad x = -1 \quad x = 1$

2) Find the zeros of $f(x) = x^3 + 3x^2 - x - 3$

Possible Rational Roots: $\pm 3 \pm 1$

Use synthetic division to test your possible roots:

$x = 1$

$$\begin{array}{c|cccc} 1\rfloor & 1 & 3 & -1 & -3 \\ & & 1 & 4 & 3 \\ \hline & 1 & 4 & 3 & 0 \checkmark \end{array}$$

$x^2 + 4x + 3$

$(x+1)(x+3)(x-1)$

Factored Form: $(x-4)(x+1)(x-1)$

Zeros: $4, -1, 1$

Factored Form: $(x+1)(x+3)(x-1)$

Zeros: $-1, -3, 1$

3) Find the rational zeros of
$$f(x) = x^3 - x^2 + 3x + 5$$

Possible Rational Roots: $\pm 5 \quad \pm 1$

Use synthetic division to test your possible roots:

```
 1 | 1  -1   3   5
   |     1   0   3
   ----------------
     1   0   3   X
```

$x=-1$
```
-1 | 1  -1   3   5
   |    -1   2  -5
   ----------------
     1  -2   5   0 ✓
```

$$x^2 - 2x + 5 = 0$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)} = \frac{2 \pm \sqrt{4-20}}{2}$$

$$\frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

Factored Form: $(x-1)(x^2-2x+5)$

Zeros: $1, 1 \pm 2i$

4) Find the rational zeros of
$$\frac{\pm 3, \pm 1}{\pm 2 \pm 1}$$

$$f(x) = 2x^3 - 3x^2 - 8x - 3$$

Possible Rational Roots: $\pm 3 \pm 1 \pm \frac{3}{2} \pm \frac{1}{2}$

Use synthetic division to test your possible roots:

```
 1 | 2  -3  -8  -3
   |     2  -1
   ----------------
     2  -1
```

```
-1 | 2  -3  -8  -3
   |    -2   5   3
   ----------------
     2  -5  -3   0 ✓
```
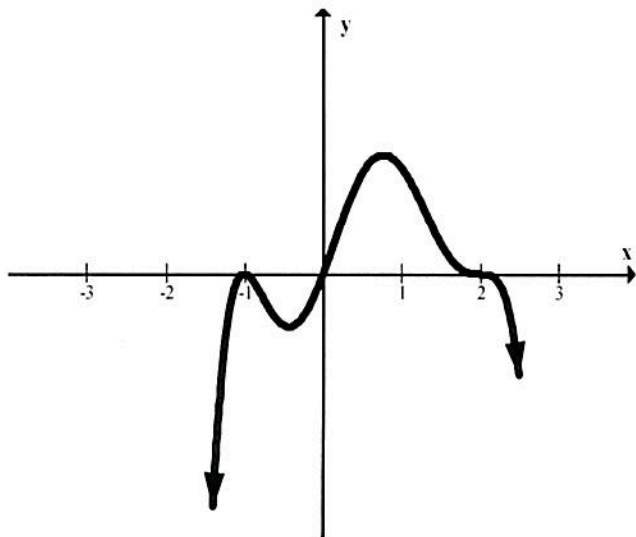
$$2x^2 - 5x - 3 = 0$$
$$(2x+1)(x-3) = 0$$

Factored Form: $(x+1)(2x+1)(x-3)$

Zeros: $-1, -\frac{1}{2}, 3$

5. A complete graph of a polynomial is shown. a) Is the degree even or odd? b) Is the leading coefficient positive or negative? c) What are the real zeros? d) What is the smallest possible degree? e) Write an equation for the given function.

a) even or odd

b) positive or negative

c) Zeros: $-1, 0, 2$

d) Degree: x^6

e) Equation $-x(x+1)^2(x-2)^3$