

Solving Quadratic Equations by Square Root Property

When $x^2 = a$, where a is a real number, then your $x = \pm\sqrt{a}$

| | | |
|------------------|-------------------|------------------------|
| <i>Examples:</i> | $x^2 - 9 = 0$ | $y^2 + 3 = 28$ |
| | $x^2 - 9 = 0$ | $y^2 + 3 - 3 = 28 - 3$ |
| | $x^2 = 9$ | $y^2 = 25$ |
| | $x = \pm\sqrt{9}$ | $y = \pm\sqrt{25}$ |
| | $x = \pm 3$ | $y = \pm 5$ |

Solving Quadratic Equations by Factoring

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0 \quad \leftarrow \text{Factoring } x$$

| | | |
|-------------|-------------|--|
| $x - 3 = 0$ | $x - 2 = 0$ | \leftarrow Set it equal to 0 and solve for x |
| $x = 3$ | $x = 2$ | |

$$1 + \frac{2}{x} - \frac{8}{x^2} = 0 \quad \leftarrow \text{Rewrite in standard form by multiplying each side of the equation by } x^2$$

$$x^2 + 2x - 8 = 0$$

$$(x + 4)(x - 2) = 0$$

| | |
|-------------|-------------|
| $x + 4 = 0$ | $x - 2 = 0$ |
| $x = -4$ | $x = 2$ |

Solution Using the Quadratic Formula

Factoring is useful only for those quadratic equations which have whole numbers. When you encounter quadratic equations that can not be easily factored out, use the quadratic formula to find the value of x :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples:

| | |
|--------------------|---|
| $x^2 - 8 = -2x$ | |
| $x^2 + 2x - 8 = 0$ | \leftarrow Rewrite in standard form, where $a = 1$, $b = 2$, and $c = -8$ |

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-8)}}{2(1)} \quad \leftarrow \text{Plug in numbers into the equation}$$

$$= \frac{-2 \pm \sqrt{36}}{2(1)}$$

$$= \frac{-2 \pm 6}{2}$$

$$= 2, -4 \quad \leftarrow \text{The two rational solutions}$$

Solution by Completing the Square

One more method of solving quadratic equations is by completing the square.

$$x^2 + 6x = -5$$

$$x^2 + 6x + (3)^2 = -5 + (3)^2$$

$$x^2 + 6x + (3)^2 = -5 + 9$$

$$x^2 + 6x + (3)^2 = 4$$

$$(x+3)^2 = 4$$

$$x+3 = \pm 2$$

$$x+3 = 2 \quad x+3 = -2$$

$$x = 2 - 3 \quad x = -2 - 3$$

$$x = -1 \quad x = -5$$

COMPLETE THESE EXERCISES FOR PRACTICE

1. $x^2 - 2x + 1 = 0$

2. $3x^2 - 5x - 12 = 0$

3. $x^3 + 3x^2 - 28x = 0$

4. $4x^2 - 12x = 16$

5. $x^2 + 3x = 0$

6. $3 + \frac{1}{x} = \frac{10}{x^2}$

7. $9y^2 + 6y - 8 = 0$

8. $y^2 - 25 = 0$

9. $8x = x^2$

10. $(x - 5)(x + 8) = -20$

11. $4 - \frac{1}{x} = \frac{3}{x^2}$

12. $2x^2 - 3x - 7 = 0$

13. $x^2 + 2x = 0$

14. $2x^2 + 4x - 8 = 0$

15. $x^3 - 2x^2 + 3x - 6 = 0$