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Inverse Functions

If the point (a, b) lies on the graph of f(x), then the point (b, a) must lie on the graph of $f^{-1}(x)$.

Two functions are inverses of each other if:

$f(f^{-1}(x)) = x$ AND $f^{-1}(f(x)) = x$

$f(x)$

x	$f(x)$
1	5
2	7
3	9
4	11

Domain: {1,2,3,4}

Range: {5,7,9,11}

$f^{-1}(x)$

x	$f^{-1}(x)$
5	1
7	2
9	3
11	4

Domain: {5,7,9,11}

Range: {1,2,3,4}

To find the inverse equation of f(x) algebraically:

1) Replace f(x) with y.
2) Interchange the x and y.
3) Solve the new equation for y.
4) New equation will be the inverse equation, $f^{-1}(x)$.

1) Find the inverse of ~~f(x)~~ $y = 8x + 3$.

$x = 8y + 3$

$-3 \quad -3$

$\frac{x-3}{8} = \frac{8y}{8}$

$f^{-1}(x) = \frac{x-3}{8}$ or $f^{-1}(x) = \frac{1}{8}x - \frac{3}{8}$

2) Which of these functions is the inverse of $f(x) = 7x + 4$?

$g(x) = \frac{x-4}{7}$ OR $h(x) = \frac{x-7}{4}$

$x = 7y + 4$

$-4 \quad -4$

$\frac{x-4}{7} = \frac{7y}{7}$ $\quad y = \frac{x-4}{7}$

3) Find the inverse of $f(x) = 3x^2 + 4$? Is the inverse a function?

$x = 3y^2 + 4$

$-4 \quad -4$

$\frac{x-4}{3} = \frac{3y^2}{3}$

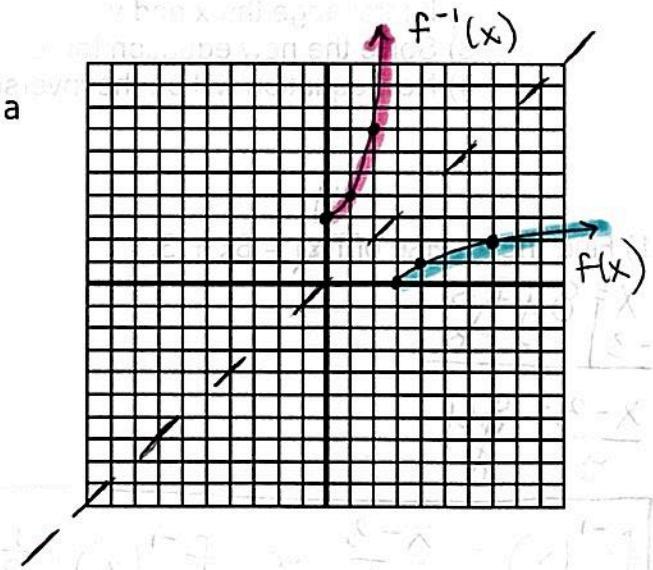
$\sqrt{y^2} = \pm\sqrt{\frac{x-4}{3}}$

$y = \pm\sqrt{\frac{x-4}{3}}$ → not a function

The graph of $f^{-1}(x)$ is a reflection of the graph of f(x) over the line y = x.
In other words, the line y = x is the line of symmetry for a function and its inverse.

4) Sketch the graphs of the inverse functions $f(x) = \sqrt{x-3}$ and $f^{-1}(x) = x^2 + 3, \ x \geq 0$ on the same coordinate system, and show that the graphs are reflections of each other in the line y = x.

(draw the line y = x as a dotted line to show symmetry)



One-to-One Functions

- Must pass the horizontal and the vertical line tests.
- Will always be increasing or always be decreasing.
- Will not have any repeated y's.
- If a function is one-to-one, its inverse will be a function.

5) Find the inverse of $f(x) = \frac{3-x}{2}$, and then graph both equations.

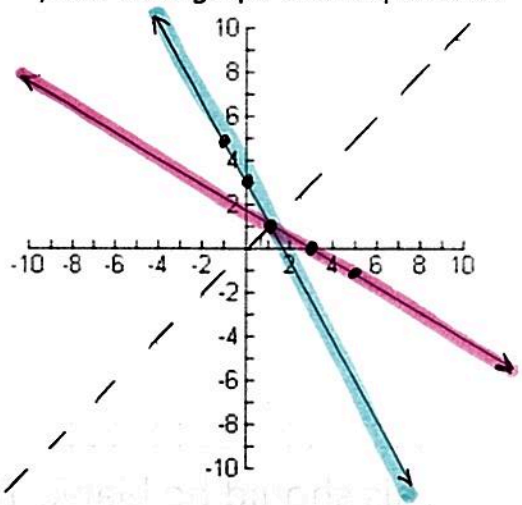
$x = \frac{3-y}{2}$ (2)

$2x = 3 - y$

$-3 \quad -3$

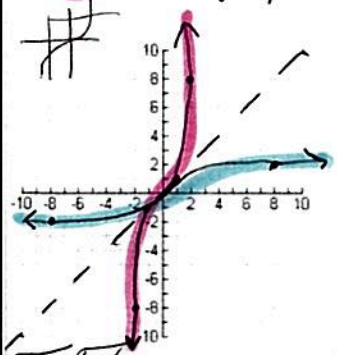
$\frac{2x-3}{-1} = \frac{-y}{-1}$

$f^{-1}(x) = -2x + 3$



6) Are the given functions one-to-one? Do they have an inverse function? If so, find the inverse.

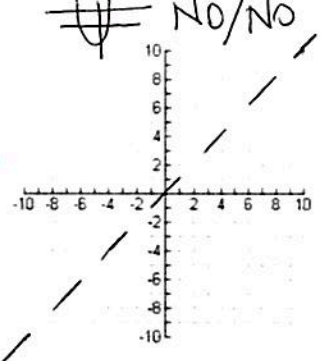
a) $f(x) = x^3$ yes/yes



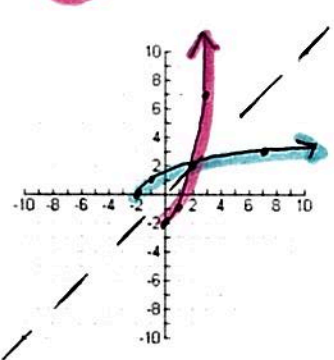
$\sqrt[3]{x} = \sqrt[3]{y^3}$

$f^{-1}(x) = \sqrt[3]{x}$

b) $f(x) = x^2 - 2$ NO/NO



c) $f(x) = x^2 - 2, x \geq 0$ yes/yes



$x = y^2 + 2$

$+2 \quad +2$

$\sqrt{x+2} = \sqrt{y^2}$

$f^{-1}(x) = \sqrt{x+2}, \ x \geq -2$